

Latent Hardening in Copper and Copper Alloys

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Keywords: crystal plasticity, latent hardening, work hardening

Abstract. A new phenomenological latent hardening model is developed for rate-dependent single crystal plasticity. The model quantitatively predicts the latent hardening evolution and latent hardening material dependence for f.c.c. single crystals. Increased overshoot, typically observed in copper alloys as opposed to copper, is rationalized based on the history dependence of latent hardening.

Introduction

Latent hardening refers to the influence of the active slip systems on hardening of the inactive slip systems and is an important concept in crystal plasticity as it controls the shape of the single crystal yield surface. Latent hardening is most commonly probed by measuring a latent hardening ratio (LHR). The LHR is obtained by dividing the initial critical shear strength after a strain path change by the maximum critical shear strength prior to the strain path change. Only one slip system is to be activated during the secondary test. Latent hardening can also be quantified by the amount of tensile axis overshoot across the symmetry boundary of the inverse standard triangle. An overshoot parameter, β_c , is defined as the ratio, at the end of overshoot, of the resolved shear stresses of the conjugate and primary systems. This measure of latent hardening corresponds to a single point in the deformation history, that at which the activity of the secondary system equals the activity of the primary system.

Careful experimental measurements indicate that the LHRs evolve with deformation [1,2]. In a well-annealed f.c.c. single crystal all systems initially have the same strength and therefore the LHRs are initially equal to one. For crystals initially oriented for single glide, the LHRs reach a maximum and then decay to a saturation value as other slip systems become active. Figure 1 shows this transition for the copper glissile system. The rate of hardening, plotted on the secondary y-axis, confirms the aforementioned evolution, i.e. the LHRs decay as the transition to stage II proceeds. In multiple glide, the LHRs quickly saturate. These trends hold for pure f.c.c. metals [1,2,3,4]. Existing latent hardening data for copper alloy single crystals [5,6,7] indicate that the LHRs, when plotted against $\tau_c^{(k)} - \tau_o$, appear to fall on the same curve as those for pure copper (see Fig. 20 in [8]). However, copper and copper alloys exhibit marked differences with respect to overshoot; copper alloys overshoot more than copper. For example, at room temperature, β_c is ~ 1.1 for copper [9,10] while β_c is ~ 1.3 for Cu-30%Zn [9,11].

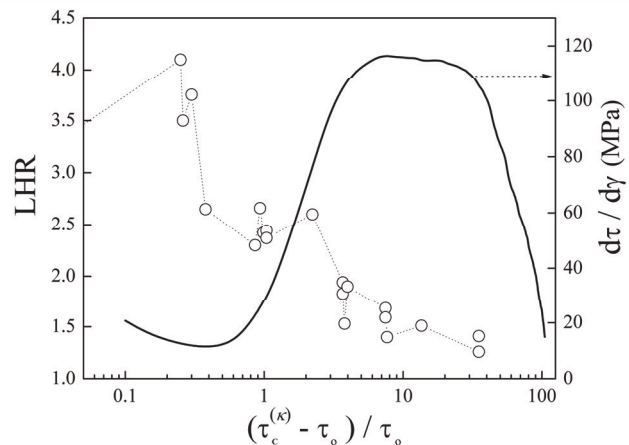


Fig. 1 Evolution of the LHR for the copper glissile system [1,2] as compared to the rate of hardening [8].

A new latent hardening model for rate-dependent crystal plasticity is formulated. The model implements latent hardening coefficients that evolve with deformation, as well as self-hardening moduli that consider the details of the transition from easy glide (stage I) to athermal hardening (stage II). Differences between copper and copper alloy latent hardening are investigated with the model and then compared to experimental latent hardening data.

Model Development

Latent hardening is implemented in crystal plasticity through the evolution equations for the critical slip system shear strength, $\tau_c^{(\alpha)}$. The Hill-Mandel form [12] of the hardening modulus is usually assumed, that is $\tau_c^{(\alpha)}$ evolves according to $\dot{\tau}_c^{(\alpha)} = h^{(\alpha\kappa)} |\dot{\gamma}^{(\kappa)}|$ where $\dot{\gamma}^{(\kappa)}$ is the shearing rate and $h^{(\alpha\kappa)}$ is the hardening modulus. The hardening modulus can be expressed as a latent hardening coefficient, $\tilde{q}^{(\alpha\kappa)}$, multiplied by a self-hardening modulus, $h^{(\kappa)}$ (no summation over κ) where the diagonal terms have $\tilde{q}^{(\alpha\kappa)} = 1$.

In the current work, the latent hardening coefficients for the non-diagonal components of $h^{(\alpha\kappa)}$ are modeled to describe the observed path dependence of the LHRs. Internal variables appropriate for the evolution equations for $\tilde{q}^{(\alpha\kappa)}$ are the critical primary shear strength and the maximum and saturation LHRs, $q_m^{(\alpha\kappa)}$ and $q_s^{(\alpha\kappa)}$, where $q_m^{(\alpha\kappa)} \geq q_s^{(\alpha\kappa)}$. The latent hardening coefficient is first separated into non-constant, $\tilde{q}_m^{(\alpha\kappa)}$, and constant, $\tilde{q}_s^{(\alpha\kappa)}$, components. In single glide, the latent critical shear stress rate can be expressed in terms of the primary critical shear stress rate as,

$$\dot{\tau}_c^{(\alpha)} = \tilde{q}^{(\alpha\kappa)} \dot{\tau}_c^{(\kappa)} = (\tilde{q}_m^{(\alpha\kappa)} + \tilde{q}_s^{(\alpha\kappa)}) \dot{\tau}_c^{(\kappa)}, \quad (1)$$

where the underscore denotes easy glide, i.e. a single slip system κ is active, $\alpha = 1, n$ and $\alpha \neq \kappa$ is applied to Eqns. (1) through (4), and n is the total number of slip systems. A suitable decay function for $\tilde{q}_m^{(\alpha\kappa)}$ is,

$$\tilde{q}_m^{(\alpha\kappa)} = Z^{(\alpha\kappa)} \operatorname{sech}^2 [c'(\tau_c^{(\kappa)} - \tau_o)], \quad (2)$$

where $\kappa = 1, n$, no summation over κ is implied, $c' = 2/(\tau_I - \tau_o)$ and τ_o and τ_I are the initial and back-extrapolated critical yield stress in shear for stage I. The latent hardening coefficients are assumed to saturate to $\tilde{q}_s^{(\alpha\kappa)} = q_s^{(\alpha\kappa)}$ at τ_I . In single glide Eqn. (1) can be integrated, via substitution of Eqn. (2), to obtain the critical shear stress on the latent systems as,

$$\tau_c^{(\alpha)} = \tau_o + c'^{-1} Z^{(\alpha\kappa)} \tanh [c'(\tau_c^{(\kappa)} - \tau_o)] + q_s^{(\alpha\kappa)} (\tau_c^{(\kappa)} - \tau_o). \quad (3)$$

Furthermore, $q_m^{(\alpha\kappa)} = \tau_c^{(\alpha)} / \tau_I$ is assumed and used to obtain $Z^{(\alpha\kappa)}$ from Eqn. (3). This leads to a general formulation for the latent hardening coefficients, viz. Eqn. (4).

$$\tilde{q}^{(\alpha\kappa)} = c' [\tau_I (q_m^{(\alpha\kappa)} - q_s^{(\alpha\kappa)}) + \tau_o (q_s^{(\alpha\kappa)} - 1)] \operatorname{sech}^2 [c'(\tau_c^{(\kappa)} - \tau_o)] + q_s^{(\alpha\kappa)}, \quad (4)$$

where $\kappa = 1, n$ and no summation over κ is implied. To complete the description of the hardening modulus an equation for $h^{(\kappa)}$ is needed. In the present work, only stages I and II are considered in the description of the self-hardening modulus and therefore $h^{(\kappa)} = h_I^{(\kappa)} + h_{II}^{(\kappa)}$. The stage I modulus is given by,

$$h_I^{(\kappa)} = (h_o - h_I) \operatorname{sech}^2 [h' \gamma^{(\kappa)}] + h_I, \quad (5)$$

where $h' = (h_o - h_l)/(\tau_l - \tau_o)$, and h_o and h_l are the initial and nominal hardening rates during stage I. Secondary slip is assumed to activate stage II, and therefore the stage II modulus is given by,

$$h_{II}^{(\kappa)} = \begin{cases} \chi^{(\kappa\beta)} \tanh^2 \left(n_a (\gamma^{(\beta)} - \gamma_{II}) / \gamma_o \right) & \text{for } \gamma^{(\beta)} > \gamma_{II} \\ 0 & \text{for } \gamma^{(\beta)} \leq \gamma_{II} \end{cases}, \quad (6)$$

where $\beta = 1$, n , $\kappa \neq \beta$, $\chi^{(\kappa\beta)}$ is the amplitude of the dislocation interaction strengthening between slip systems κ and β , γ_{II} represents the amount of secondary slip necessary for the initial activation of stage II, n_a is the number of active slip systems, and $\gamma_o / n_a + \gamma_{II}$ is amount of slip after which the interaction reaches ~60% of peak strength. A slip system is considered active, with regards to the stage II transition, if the fractional shearing rate is greater than a critical amount, i.e.,

$$n_a = \sum_{\alpha=1}^n n_a^{(\alpha)}, \quad n_a^{(\alpha)} = \begin{cases} 1 & \text{for } \dot{\gamma}^{(\alpha)} / \sum \dot{\gamma}^{(\beta)} \geq 0.1 \\ 0 & \text{for } \dot{\gamma}^{(\alpha)} / \sum \dot{\gamma}^{(\beta)} < 0.1 \end{cases}, \quad (7)$$

where $\beta = 1$, n . Further details relating to the new crystal plasticity hardening law can be found in [13]. A rate-dependent finite element implementation of the crystal plasticity constitutive law, as described in [14], is followed. The slip system shearing rate is given by a power law representation,

$$\dot{\gamma}^{(\alpha)} = \dot{\gamma}_o^{(\alpha)} \left(\tau^{(\alpha)} / \tau_c^{(\alpha)} \right)^{1/m} \text{sgn} \left(\tau^{(\alpha)} \right) \quad (8)$$

where m is the strain rate sensitivity and $\dot{\gamma}_o^{(\alpha)}$ a reference strain rate. A tension test for a single crystal oriented with the tensile axis parallel to [3 2 1] is simulated. Primary and conjugate slip are predominantly activated for this test configuration. The interaction between these slip systems can be classified as sessile [8] and therefore the results presented are for the sessile interaction system. Results for the sessile system are compared to experimental data for the glissile system; this comparison is justified as sessile and glissile interaction systems exhibit equivalent latent hardening behaviour [8]. Superscripts for results pertaining to sessile interactions are hereafter dropped, while superscripts for results of a general nature are retained. Copper is chosen for the simulation due to the availability of experimental latent hardening data for both copper and copper alloys. Material parameters for copper are shown in Table 1, with units, where appropriate, in MPa. Solid solution strengthening is accounted for by increasing τ_o and τ_l and by adjusting γ_{II} where necessary. All other parameters, to a first approximation, are considered unchanged by alloying.

Table 1. Material parameters for copper

| $1/m$ | $\dot{\gamma}_o$ | h_o | h_l | τ_o | τ_l | q_m | q_s | χ | γ_o | γ_{II} |
|-------|------------------|-------|-------|----------|----------|-------|-------|--------|------------|---------------|
| 150 | 0.001 | 60.0 | 10.0 | 0.80 | 1.00 | 3.50 | 1.00 | 90.0 | 0.001 | 0.000 |

Results and Discussion

Most existing phenomenological crystal plasticity constitutive laws employ constant latent hardening coefficients; $\tilde{q}_m^{(\alpha\kappa)} = 0$ is assumed. In the present notation, diagonal latent hardening is defined by $\tilde{q}_s^{(\alpha\kappa)} = \delta^{(\alpha\kappa)}$ where $\delta^{(\alpha\kappa)}$ is the Kronecker delta, isotropic latent hardening is defined by

$\tilde{q}_s^{(\alpha\kappa)} = 1$ and multi-parameter latent hardening is defined by $\tilde{q}_s^{(\alpha\kappa)} = q_s^{(\alpha\kappa)}$ where $q_s^{(\alpha\kappa)}$ is typically chosen to be between 1.0 and 1.4. The difference between constant and variable latent hardening coefficients, as formulated in the new law, is shown by plotting the latent shear stress as a function of the primary shear stress for the different latent hardening frameworks defined above (Fig. 2). The use of constant latent hardening coefficients leads to LHRs that evolve from a value of one to a saturation value. The new latent hardening law leads to LHRs that evolve from a value of one, reach a maximum at τ_l and then decay to a saturation value. A better fit of the experimental LHR data can be obtained by using the new formulation, as shown for a copper single crystal with an initial easy glide orientation (Fig. 3).

The apparent discrepancy between the LHRs and overshoot as observed for copper and its alloys is explored with the new latent hardening formulation. Latent hardening evolves according to the association between $\tilde{q}^{(\alpha\kappa)}$ and $h^{(\kappa)}$ and therefore attention is focused on parameters that influence self-hardening as well as those directly associated with the latent hardening coefficients.

Both q_m and q_s influence overshoot, however, for the range of values within the experimental limits for copper, q_s has a greater influence. A value of $q_s=1.4$, as used in Fig. 2, is usually considered appropriate for copper. However, the use of $q_s=1.0 - 1.1$ leads to β_c values consistent with pure copper, whereas $q_s=1.4$ leads to a β_c value consistent with alloys such as Cu-30%Zn or Cu-10%Al, for instance. These trends are shown in Fig. 4 where the ratio of the conjugate and primary resolved shear stresses provides a measure of angular distance between the tensile axis and symmetry boundary and hence overshoot. Furthermore, a linear regression of the compiled LHR data for copper and copper alloys reveals that q_s is between 1.0 and 1.1 [8]. Therefore, a low value for q_s , between 1.0 and 1.1, is considered most appropriate for copper; a LHR of 1.4 merely represents an earlier stage of deformation. The results also suggest that q_s should increase with increasing alloy content (Fig. 4). Such a trend has never been experimentally documented, and therefore an alternate explanation to account for the increased overshoot observed in alloys is proposed based on association between latent and self-hardening.

Increased overshoot is usually observed in alloys that exhibit extended easy glide strain and solid solution strengthening [8,15,16]. It is therefore relevant to explore, within the context of the model, how extended easy glide and solid solution strengthening influence overshoot. The trend, observed for pure metals, of increasing easy glide strain with decreasing stacking fault energy is captured with

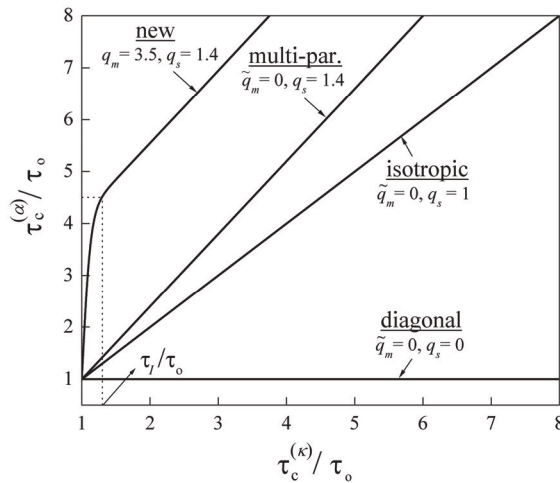


Fig. 2 Latent versus the primary shear stress for different latent hardening frameworks described in the text.

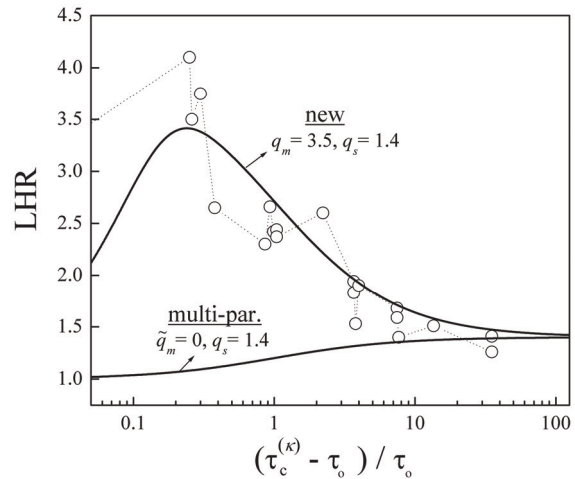


Fig. 3 Latent hardening ratio for copper glissile system comparing experiment [1,2] to simulation.

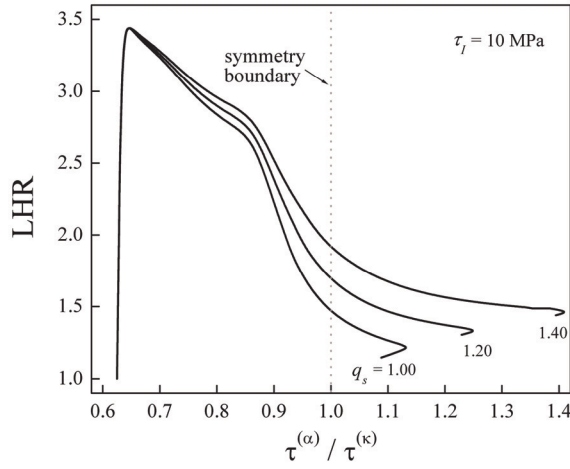


Fig. 4 Latent hardening ratio versus ratio of the resolved shear stress on the primary and conjugate systems for different q_s values.

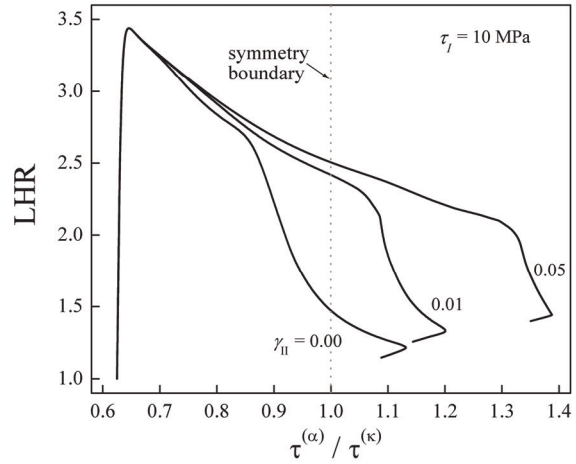


Fig. 5 Latent hardening ratio versus ratio of the resolved shear stress on the primary and conjugate systems for different γ_{II} values.

the model by using appropriate values for q_m , i.e. by scaling q_m with reduced stacking fault energy [7]. However, the extended easy glide strain associated with alloys cannot be captured by using realistic latent hardening coefficients. Instead, increased easy glide strain is in part achieved by adjusting γ_{II} . Figure 5 shows the LHR evolution and overshoot behaviour when γ_{II} is varied from 0.00 to 0.05 with all other material parameters held constant. A significant increase in the amount of easy glide strain is observed. As a consequence, a high LHR persists to much larger strains and this leads to an increased amount of overshoot, e.g. β_c increases from 1.1 to 1.4 as γ_{II} is increased from 0.00 to 0.05 (Fig. 5). The influence of extended easy glide strain on overshoot is more pronounced with increasing τ_l . Similarly, overshoot is predicted to increase as τ_l increases; this trend is most pronounced when the easy glide strain is extended.

The LHR evolution curves, when plotted against $\tau_c^{(\kappa)} - \tau_o$, collapse onto a single curve as γ_{II} is varied, yet decay more gradually with increasing τ_l . This shift in the LHR evolution curve with τ_l , as shown in Fig. 6, is in opposition to the accepted trends [5,6,8]. However, a re-examination of the experimental LHR data reveals that the copper alloy LHRs are shifted to higher values of $\tau_c^{(\kappa)} - \tau_o$ and follow a distinct evolution curve, in accord with the model predictions. The difference between copper and alloyed copper is more evident if one accounts for the logarithmic scale of the x-axis.

To explain the trends in a purely phenomenological manner, it is helpful to define the maximum difference between the latent and primary shear stress as $\Delta\tau_m = \tau_l(q_m - 1)$. The LHR decay towards saturation can therefore be thought to evolve according to $\Delta\tau_m / \tau^{(\kappa)}$. As τ_l

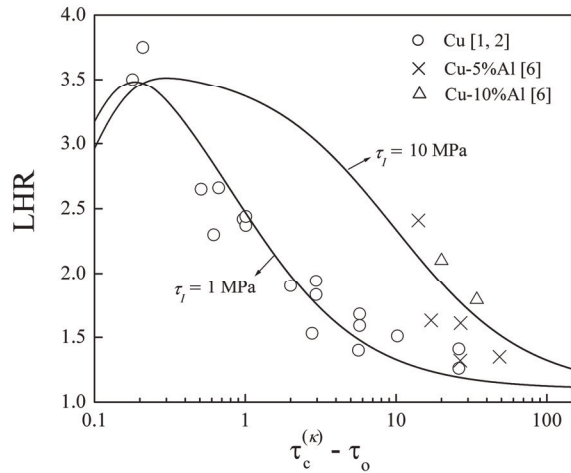


Fig. 6 Latent hardening ratio evolution for copper and copper alloys comparing experiment to simulation.

increases, a greater increase in $\tau^{(\kappa)}$ is required to achieve an equivalent LHR decay for a given q_m as $\Delta\tau_m$ is proportional to τ_l whereas work hardening is independent of τ_l . The ability to identify the correct latent hardening trends is of fundamental importance for determining the operative mechanisms controlling latent and work hardening in f.c.c. metals and alloys. The trends predicted by the model cannot be validated conclusively due to paucity of experimental data available in the literature; clearly more experimental LHR data for alloyed f.c.c. materials is needed. Nonetheless, the model predicts that overshoot observed in f.c.c. alloys can be at least partly attributed to the LHR history dependence associated with $\Delta\tau_m$ and easy glide, both of which typically increase with alloying.

Conclusions

A new crystal plasticity hardening law has been developed which implements latent hardening coefficients that evolve with deformation. The law allows for an effective means to better control the orientation and material dependence of latent hardening in rate-dependent crystal plasticity. The model has also been used to contrast the latent hardening behaviour observed in copper and copper alloys. The results lead to a new interpretation of the influence of alloying on the amount of overshoot observed in f.c.c. alloys.

Acknowledgements

This work was supported by General Motors through the GM/Brown Collaborative Research Laboratory for Computational Materials Research.

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