Dental Dams vis-à-vis Elasticity

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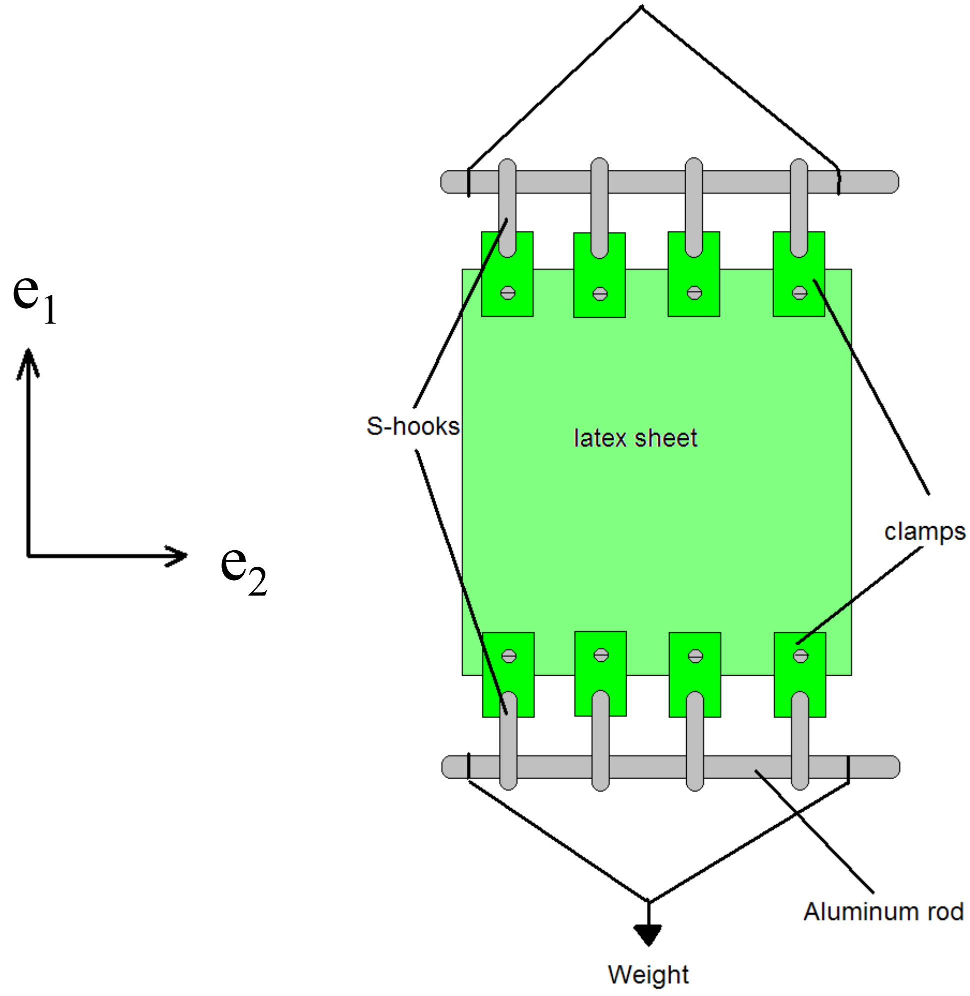
Introduction

- The latex is assumed to be isotropic.
- Unaxial stress – addresses the compressibility question and provides initial data for determining the elastic potential.
- Biaxial stress – an additional test of the elastic potential.
- Plane stress elastic potential and out of plane stretch can be found.

Uniaxial Stress Set-up

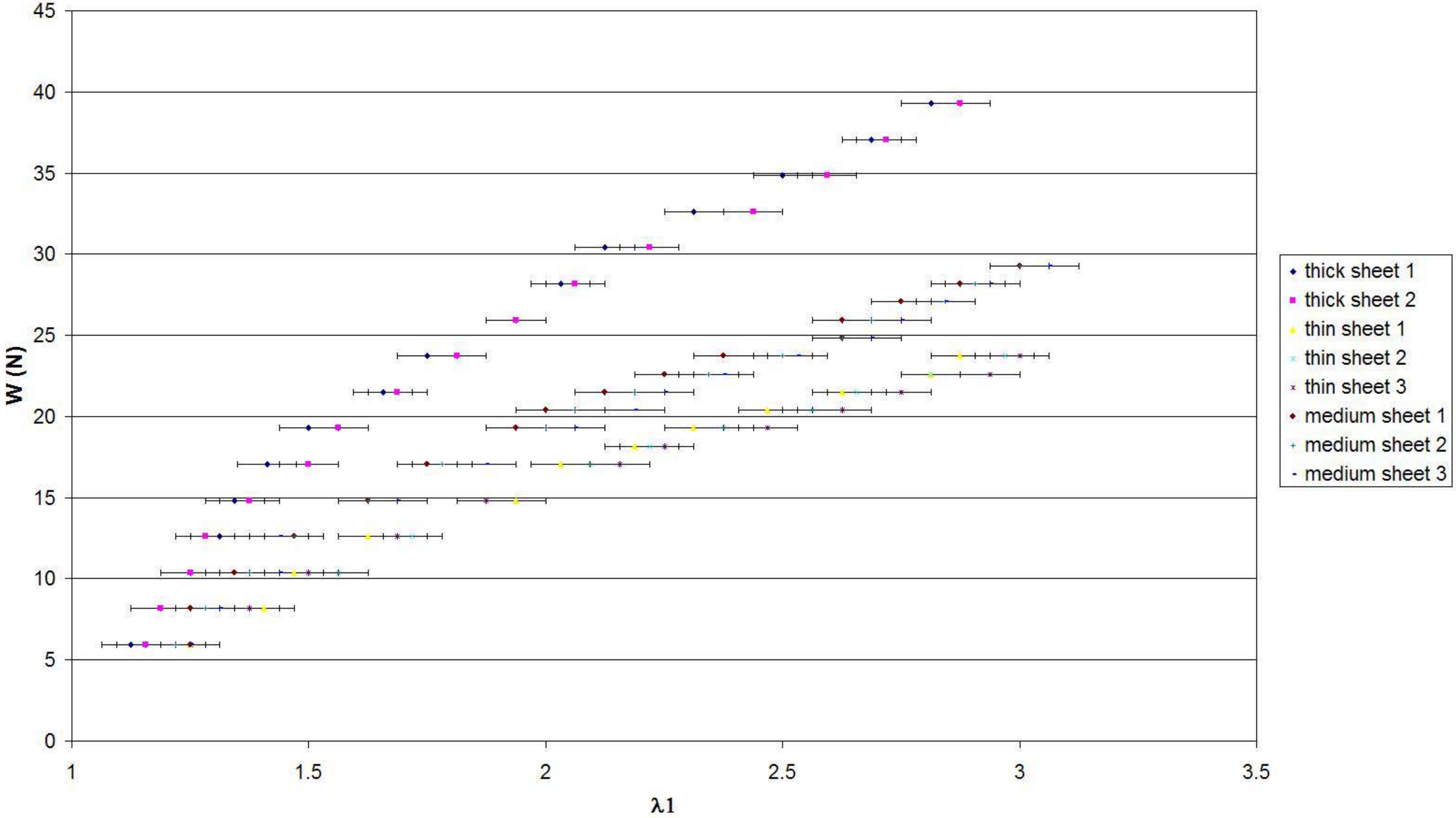


Latex sheet is incompressible.

- For an incompressible material under uniaxial stress, $\lambda_1\lambda_2^2=1$.
- From the uniaxial experiment,

$$\langle\lambda_1\lambda_2^2\rangle = 1.067 \pm 0.1$$

Uniaxial stress data: Weight vs. λ_1



Data fitted with Ogden Potential

- $P_1=(2\mu/\alpha)[\lambda_1^{\alpha-1} - \lambda_1^{-(\alpha/2+1)}]$

  $W= P_1 A$
- Thick sheet: $\alpha=1.60$, $2\mu A_{thick}=35.96$

  Medium sheet: $\alpha=1.65$, $2\mu A_{med}=24.81$

  Thin sheet: $\alpha=1.68$, $2\mu A_{thin}=20.26$
- $\langle\alpha\rangle=1.643$

Data fitted with Power Law Potential.

- $P_1=\mu(\lambda_1-\lambda_1^{-2})[1+(b/n)(\lambda_1^2+2\lambda_1^{-1}-3)]^{n-1}$

  $W= P_1 A$

- Thick sheet: did not converge

  Medium sheet: $b=0.71$, $n=0.88$, $\mu A_{med}=12.34$

  Thin sheet: $b=0.24$, $n=0.83$, $\mu A_{thin}=9.86$

- <b>=0.475, <n>=0.855

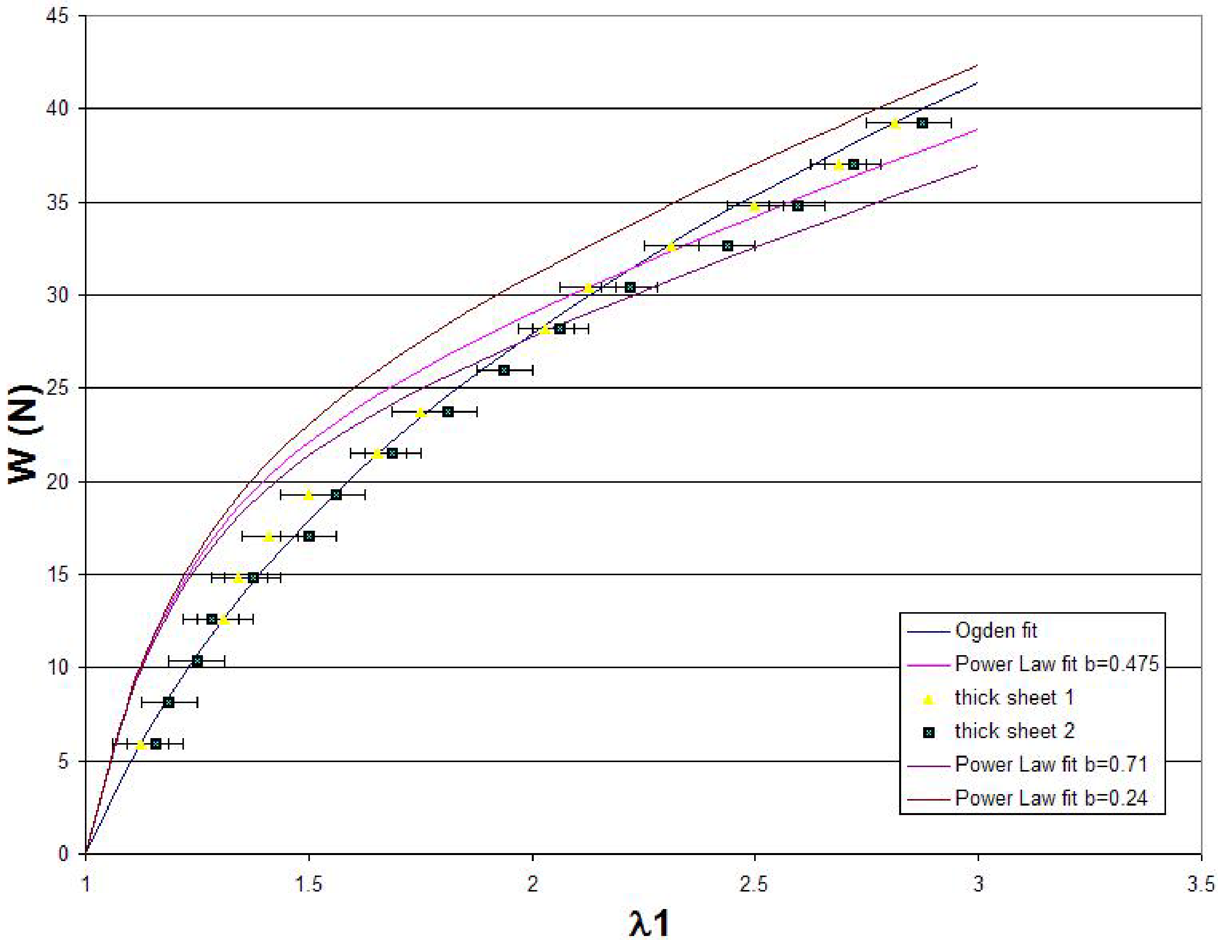
The relationship between μ and sheet thickness (h).

- Assume the Ogden fit of the thick sheet yields the proper relationship between μ and h, namely,

$$\mu=3908.7/h$$

- Since h is quite small, it is important to assign to find its exact value to ensure a reasonable value of μ.

Fits and Thick Sheet Uniaxial Data



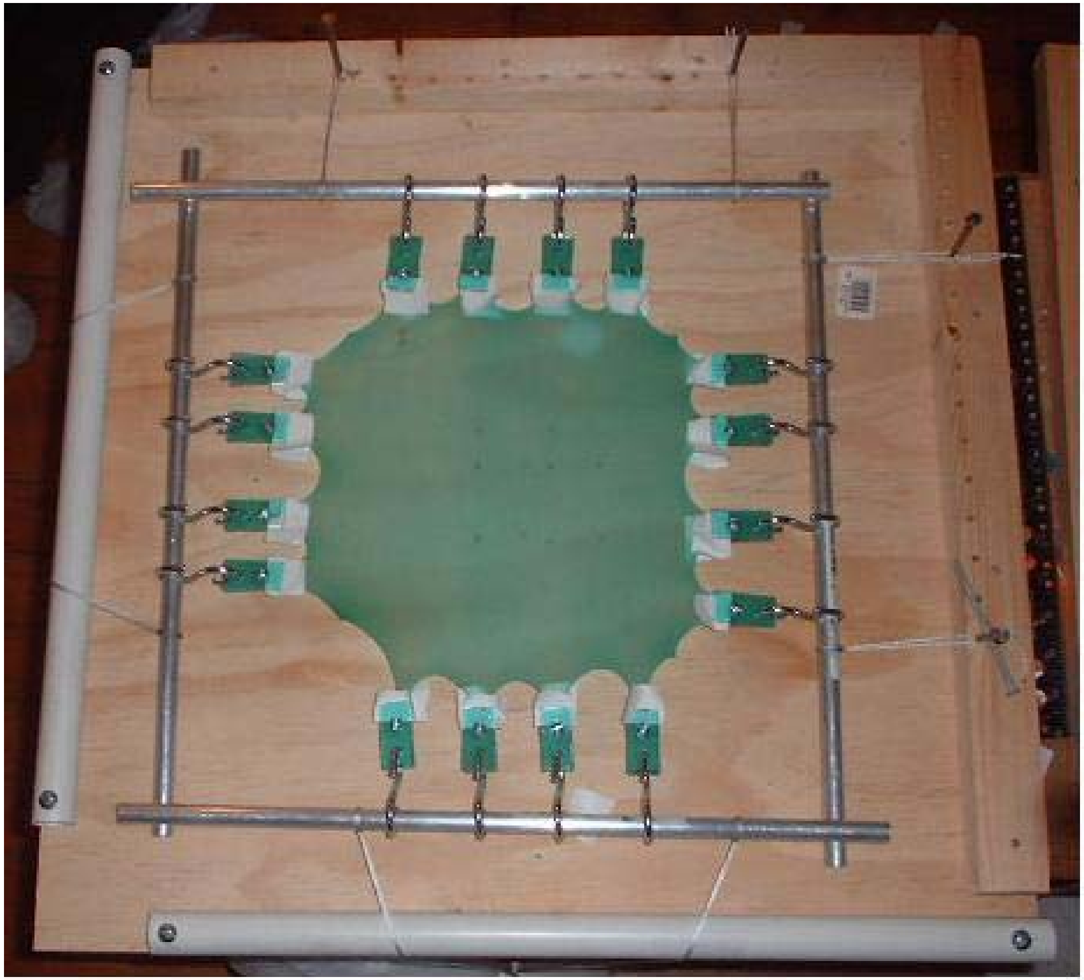
Ogden: $\alpha=1.643$, $\mu A_{thick}=17.98$

Power Law: $n=0.855$, $\mu A_{thick}=17.98$

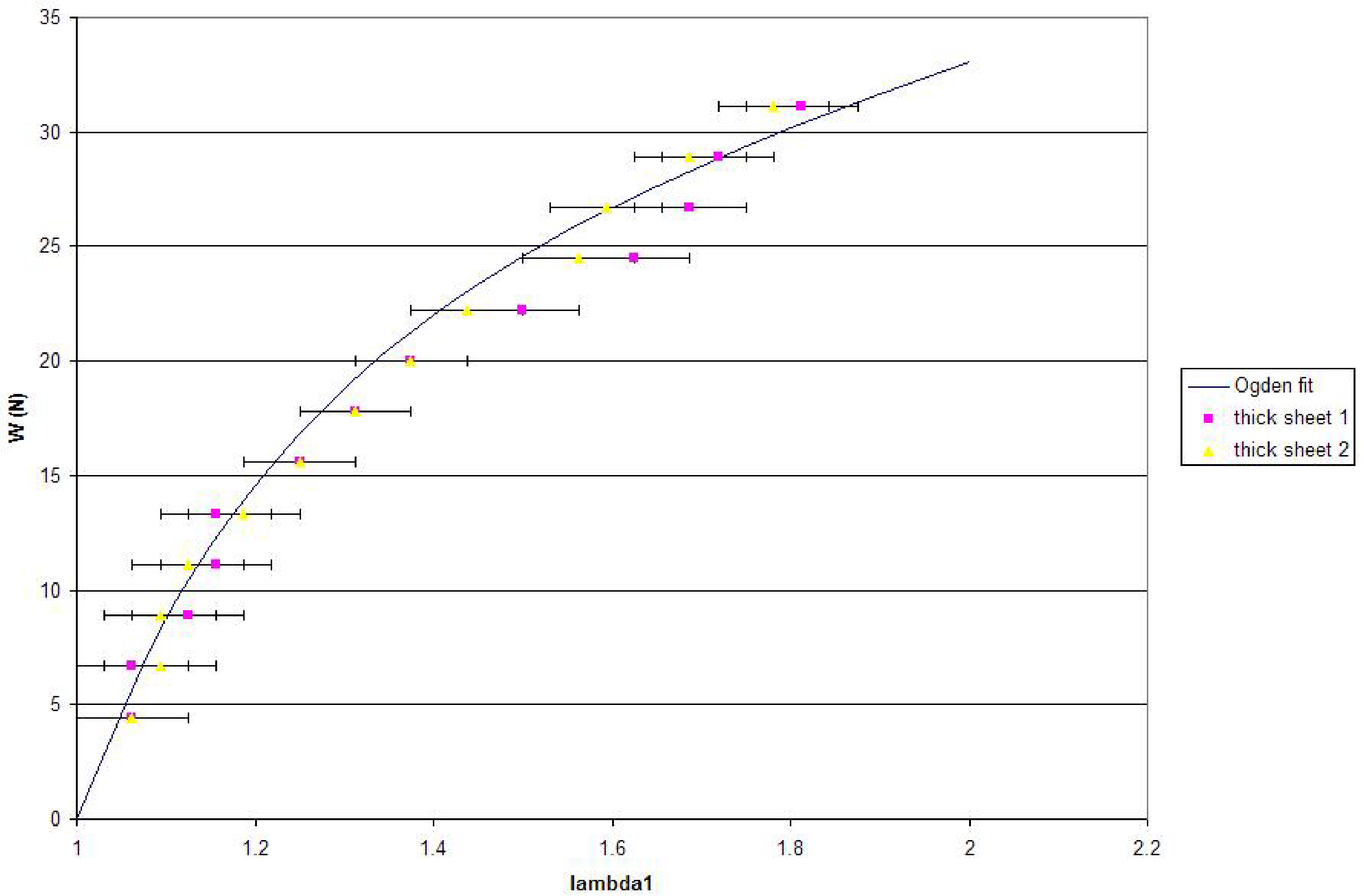
The Ogden potential fits quite well.

Let’s see how it does with Biaxial stress.

Biaxial Stress Set-up



Data and Odgen fit for Biaxial Stress



$P_1 = (2\mu/\alpha)[\lambda_1^{\alpha-1} - \lambda_1^{-(\alpha/2+1)}]$, $W = P_1 A_{thick}$

$\alpha = 1.643$, $\mu A_{thick} = 17.98$

Again, the Ogden potential does the job.

Now let's find the out-of-plane stretch and the plane stress elastic potential.

Since the material is incompressible, the out of plane stress comes from the constraint.

$$\lambda_3=1/(\lambda_1\lambda_2)$$

The plane stress potential immediately follows.

$$w=(2\mu/\alpha^2)(\lambda_1{}^{\alpha}+\lambda_2{}^{\alpha}+\lambda_3{}^{\alpha}-3)$$
$$=(2\mu/\alpha^2)(\lambda_1{}^{\alpha}+\lambda_2{}^{\alpha}+\lambda_1{}^{-\alpha}\lambda_2{}^{-\alpha}-3)$$

$$w=0.74\mu(\lambda_1{}^{1.643}+\lambda_2{}^{1.643}+\lambda_1{}^{-1.643}\lambda_2{}^{-1.643}-3)$$

Conclusion

- Uniaxial and biaxial experiments show the elastic potential of the latex has an Ogden form.
- The value of μ still needs to be determined. This can be done by obtaining an accurate value of the sheet thickness.