"Devising the Hyperelastic Potential for Thin Latex Rubber Sheets"

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EN-0227
December 3rd,2003

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Motivation

- Hyperelastic potential for material describes deformation response under different loading conditions
- Derivative of hyperelastic potential with respect to the deformation gradient is the nominal stress → relates potential to physical phenomenon that can be measured

$$P_i = \frac{\partial W}{\partial \lambda_i}$$

Where P-Nominal Stress

W-Hyperelastic Potential

λ -Principal stretch

- Thin latex rubber sheets of three different thicknesses were deformed under uniaxial and biaxial tests

1. stresses and stretches were measured
2. from this data hyperelastic potential can be devised

Uniaxial Test – “Mother of All Tests”

- easiest test to perform
- determines compressibility or incompressibility
- determines in-plane material symmetry
- provides relationship between

  longitudinal and transverse stretches
- provides a stress-stretch relation for the

  longitudinal directions

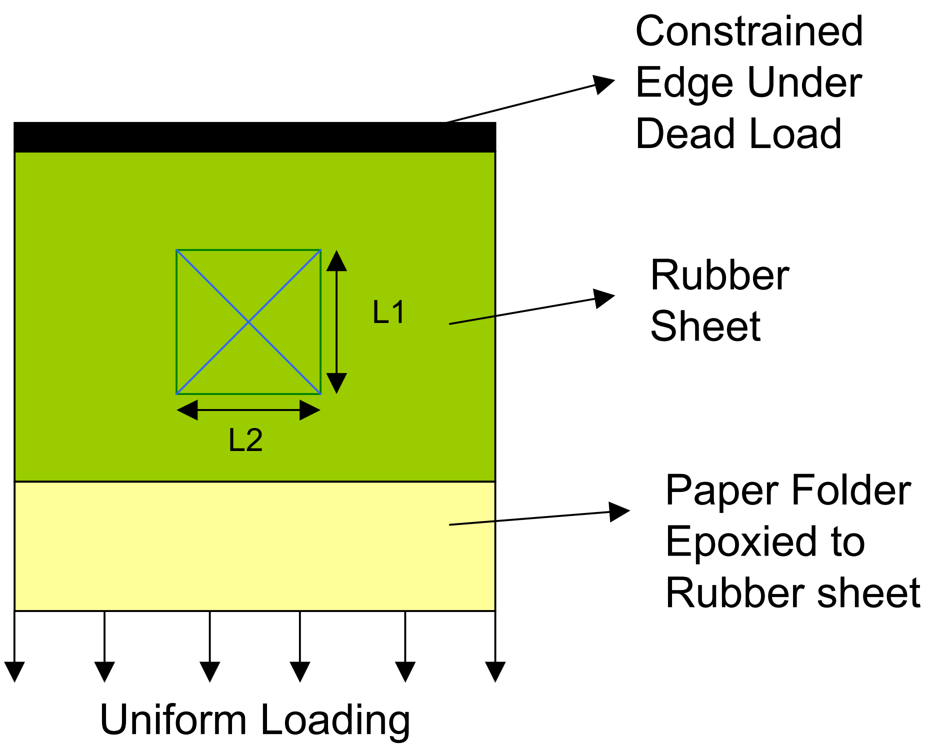
Experimental Set-up

- **Bottom edge**
  1. Epoxied to paper folder
  2. Chick pea cans(Stop&Shop) clamped to paper folder.
  3. Added more cans to increase loading.

- **Upper Edge**
  1. Rolled and epoxied to a pencil.
  2. Constrained the pencil using all the books we could find.

- **Measurements**
  1. L1 and L2 were measured for each load step using a standard ruler (**Least count=1mm**).

Repeated same procedure for all 3 samples of rubber sheet.



Biaxial Test

- not as easy as uniaxial
- provides with another in-plane stress-stretch relation (more data!!!)
- useful in conjunction with uniaxial test data in finding the Elastic Potential

Experimental Set-up

- **Set-Up was made on a smooth horizontal surface.**
- **Friction between the sheet and surface was further reduced using lubricating lotion.**

- **Loaded Edges**

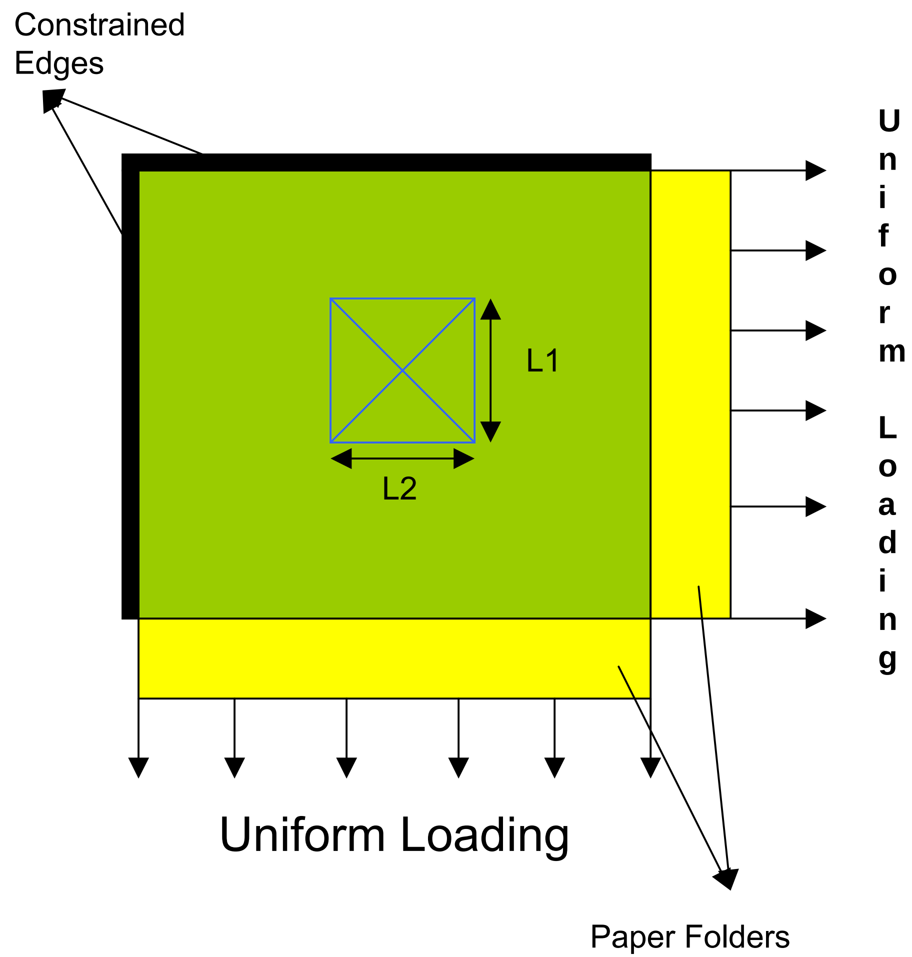
1. Paper folders were epoxied.
2. Chick pea cans(Stop&Shop) clamped to paper folder.
3. Cans were hung down on the sides of horizontal surface.
4. Added more cans to increase loading.

- **Constrained Edges**

1. Rolled and epoxied to pencils
2. Constrained using all our books and some borrowed ones.

- **Measurements**

1. L1 and L2 were measured using standard ruler (**Least count=1mm**) for each load step.



Uniaxial Test Data

Data for Thick sheet(0.22mm)

thick				
Lo=7.6cm	to=.22mm	Ao=Ltot*to=0.00003234m^2		
Bo=6.9cm		Stress=wts/Ao		
Ltot=14.7cm				
wts(kg)	stress(KPa)	L1(long)	L2(tran)	Incompressibility check(L1*L2^2)
0.5	15.46072975	1.07	0.99	1.048707
0.703	21.73778602	1.11	0.97	1.044399
1	30.92145949	1.16	0.96	1.069056
1.203	37.19851577	1.22	0.96	1.124352
1.5	46.38218924	1.3	0.93	1.12437
1.703	52.65924552	1.37	0.91	1.134497
2	61.84291899	1.5	0.9	1.215
2.203	68.11997526	1.59	0.89	1.259439
2.5	77.30364873	1.76	0.86	1.301696

- For incompressibility L1*L2*L3=1
- For Uniaxial test L2=L3
- Hence L1*L2^2= 1
- This is close to 1 for small stretches but not true for larger values
- Material is not incompressible

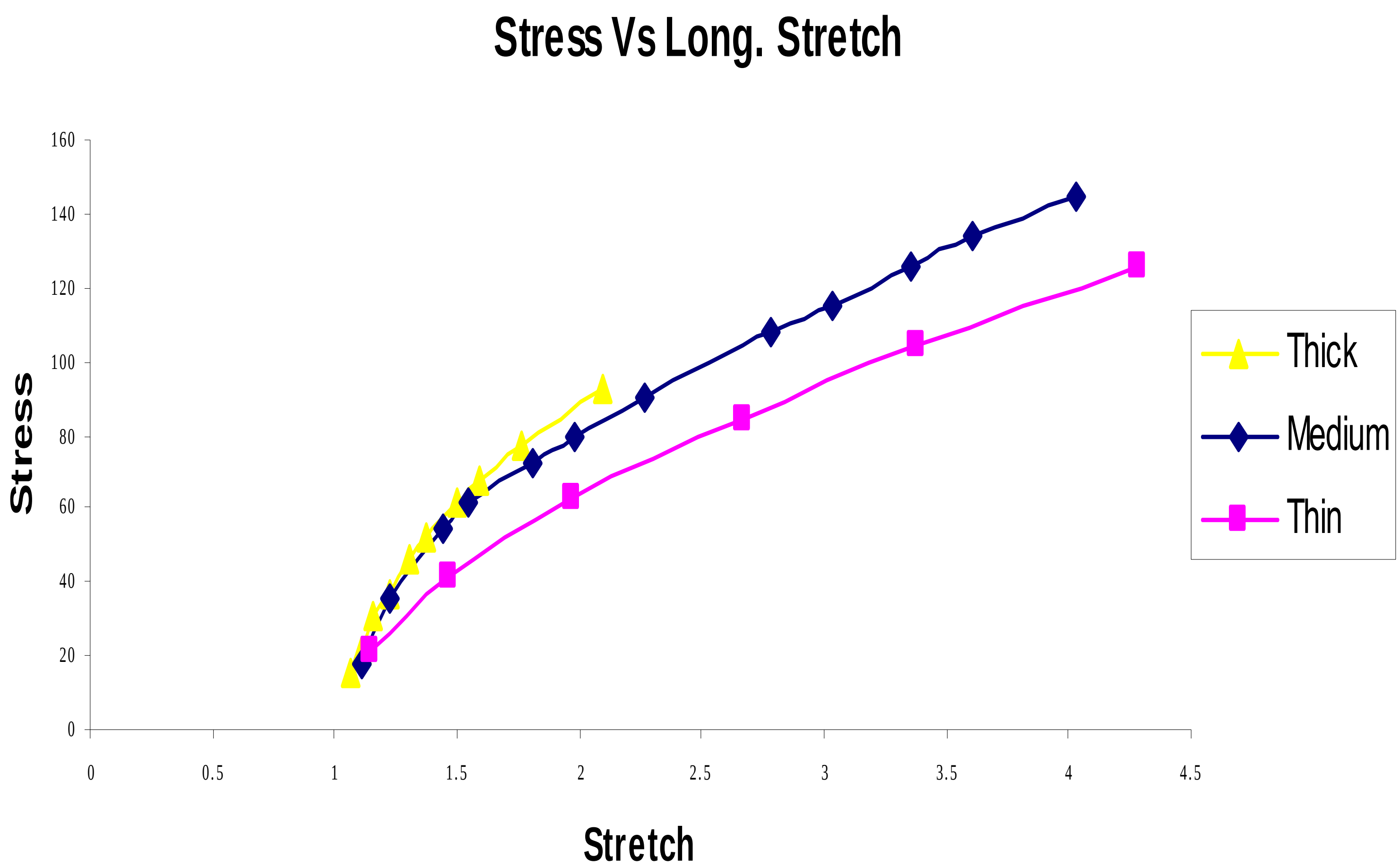
Data for medium sheet(0.19mm)

medium				
Lo=7.7cm	to=.19mm	Ao=.00002774m^2		
Bo=6.2cm				
Ltot=14.6cm				
wts(kg)	stress(KPa)	L1(Long)	L2(Tran)	Incompressibility check(L1*L2^2)
0.5	18.024	1.11	0.98	1.066044
1	36.049	1.23	0.95	1.110075
1.5	54.073	1.44	0.9	1.1664
1.703	61.391	1.55	0.89	1.227755
2	72.098	1.81	0.85	1.307725
2.203	79.416	1.98	0.82	1.331352
2.5	90.123	2.27	0.77	1.345883
3	108.147	2.78	0.71	1.401398
3.203	115.465	3.03	0.67	1.360167
3.5	126.172	3.35	0.65	1.415375
3.703	133.49	3.61	0.62	1.387684
4	144.196	4.03	0.58	1.355692

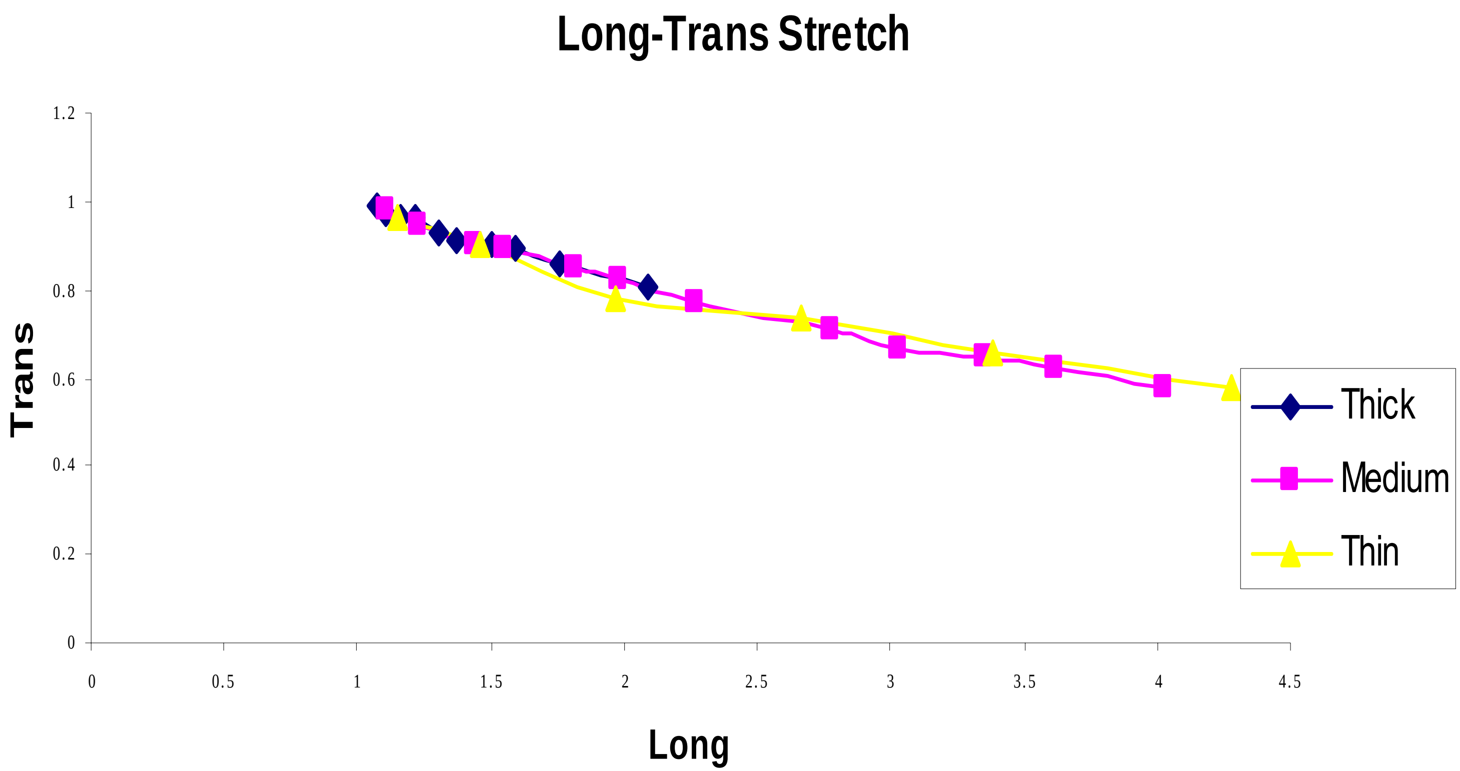
Data for thin sheet (.16mm)

thin				
Lo=7.8cm	to=.16mm	Ao=.00002384m^2		
Bo=5.0cm				
Ltot=14.9cm				
wts(kg)	stress(Kpa)	L1(long)	L2(tran)	Incompressibility check(L1*L2^2)
0.5	20.973	1.15	0.96	1.05984
1	41.946	1.46	0.9	1.1826
1.5	62.919	1.97	0.78	1.198548
2	83.893	2.67	0.74	1.462092
2.5	104.866	3.38	0.66	1.472328
3	125.839	4.28	0.58	1.439792

- Plasticity was observed in thin sheet
- Lo after unloading changed to 8.1 cm



- Thin sheet behavior is thought to be different because of:
  1. Plasticity in thin sheet
  2. Thin sheet shows more of a transversely isotropic behavior while the other two sheets behave more as homogeneously isotropic
  3. These lead to choosing a different potential for thin sheet



- Power law response for all three sheets were similar
- We chose power law for medium sheet as we had most confidence in this data set
- The power law chosen was L2=L1^(-0.4)

Assumed Potential

- We assumed a very general 3-D Hyperelastic Potential

$$W = \frac{\mu}{\beta}\left(\lambda_1^{-\beta} + \lambda_2^{-\beta} + \lambda_3^{-\beta} - 3\right) + \frac{\mu}{\alpha}\left(\left(\lambda_1\lambda_2\lambda_3\right)^{\alpha} - 1\right)$$

For a uniaxial test

$$P_1 = \frac{\partial W}{\partial \lambda_1}$$

$$P_2 = \frac{\partial W}{\partial \lambda_2} = 0 = P_3$$

Hence

$$\lambda_2 = \lambda_1^{-\left(\frac{\alpha}{\beta + 2\alpha}\right)}$$

$$P_1 = \mu\left(\lambda_1^{\frac{\alpha\beta - 2\alpha - \beta}{\beta + 2\alpha}} - \lambda_1^{-(\beta+1)}\right)$$

More Mathematics

From Power law for medium sheet

$$\frac{\alpha}{\beta + 2\alpha} = 0.4$$

Hence

$$\alpha = 2\beta$$

Hence

$$P_1 = \frac{\mu}{\lambda_1}\left(\lambda_1^{\frac{2\beta}{5}} - \lambda_1^{-\beta-1}\right)$$

Now by using *Mathematica* to fit the given data for medium sheet into the last expression, we get:

$$\beta = 4.65$$

$$\mu = 44.09$$

$$\alpha = 9.30$$

- Hence

$$W = \frac{44.09}{4.65}\left(\lambda_1^{-4.65} + \lambda_2^{-4.65} + \lambda_3^{-4.65} - 3\right) + \frac{44.09}{9.30}\left(\left(\lambda_1\lambda_2\lambda_3\right)^{9.30} - 1\right)$$

Modified Potential for Thin Sheet

2-D Potential (Only valid for thin sheets)

$$W = K\left[\frac{\mu}{\beta}\left(\lambda_1^{-\beta} + \lambda_2^{-\beta} - 2\right) + \frac{\mu}{\alpha}\left(\left(\lambda_1\lambda_2\right)^{\alpha} - 1\right)\right]$$

(K is a function of thickness)

Hence for Uniaxial test

$$P_1 = K\mu\left(\lambda_1^{\frac{\alpha\beta-\alpha-\beta}{\alpha+\beta}} - \lambda_1^{-(\beta+1)}\right)$$

$$\lambda_2 = \lambda_1^{-\frac{\alpha}{\alpha+\beta}}$$

More Mathematics

From Power law for medium sheet

$$\frac{\alpha}{\beta + \alpha} = 0.4$$

Hence

$$\alpha = \frac{2}{3}\beta$$

Hence

$$P_1 = K\frac{\mu}{\lambda_1}\left(\lambda_1^{\frac{2\beta}{5}} - \lambda_1^{-\beta-1}\right)$$

Assume exponential law for K when the thickness is less than .18mm

$$K = \exp\left[(.18 - t_o)^{\delta}\right]$$

$$\beta = 4.65$$

$$\mu = 44.09$$

$$\alpha = 3.10$$

$$\delta = 10$$

- Hence

$$W = \exp\left[(.18 - t_o)^{10}\right]\left[\frac{44.09}{4.65}\left(\lambda_1^{-4.65} + \lambda_2^{-4.65} - 2\right) + \frac{44.09}{3.10}\left((\lambda_1\lambda_2)^{3.10} - 1\right)\right]$$

Checking Potential

We calculated the loads and transverse stretches for the different

Sheets using the Potential.

Thick Sheet

wts(kg)	stress(KPa)	L1	L2	Stress Using Pot.	L2 using pot
0.5	15.46072975	1.07	0.99	16.64850353	0.973299475
0.703	21.73778602	1.11	0.97	23.78083183	0.959115277
1	30.92145949	1.16	0.96	31.03134814	0.942359915
1.203	37.19851577	1.22	0.96	37.9776823	0.92354076
1.5	46.38218924	1.3	0.93	45.23697899	0.900373406
1.703	52.65924552	1.37	0.91	50.35385653	0.881681591
2	61.84291899	1.5	0.9	58.02448341	0.850283
2.203	68.11997526	1.59	0.89	62.48684187	0.830694147
2.5	77.30364873	1.76	0.86	69.88572022	0.797617975
3	92.76437848	2.09	0.81	82.42769469	0.74463164

Medium Sheet

wts(kg)	stress(KPa)	L1	L2	Stress Using Pot	L2 Using Pot
0.5	18.024	1.11	0.98	23.78083183	0.959115277
1	36.049	1.23	0.95	38.99240227	0.920530019
1.5	54.073	1.44	0.9	54.71166092	0.864281074
1.703	61.391	1.55	0.89	60.56600755	0.839203566
2	72.098	1.81	0.85	71.8985404	0.788730382
2.203	79.416	1.98	0.82	78.4069849	0.760911108
2.5	90.123	2.27	0.77	88.80301837	0.720426463
3	108.147	2.78	0.71	106.0864034	0.664327272
3.203	115.465	3.03	0.67	114.3040945	0.641834339
3.5	126.172	3.35	0.65	124.6558719	0.616569556
3.703	133.49	3.61	0.62	132.9518316	0.598407626
4	144.196	4.03	0.58	146.1681107	0.572635122

Thin Sheet (Using Modified Potential)

wts(kg)	stress(Kpa	L1	L2	Stress Using Pot	L2 Using Pot
0.5	20.973	1.15	0.96	24.35738013	0.945629177
1	41.946	1.46	0.9	45.79927774	0.859525682
1.5	62.919	1.97	0.78	63.98894603	0.762453758
2	83.893	2.67	0.74	83.98980908	0.675142574
2.5	104.866	3.38	0.66	103.0067719	0.614374695
3	125.839	4.28	0.58	126.2298114	0.559013753

Mathematics for Biaxial Tests

$$P_1 = \frac{\partial W}{\partial \lambda_1} = P_2 = \frac{\partial W}{\partial \lambda_2} = \frac{\partial W}{\partial \lambda}$$

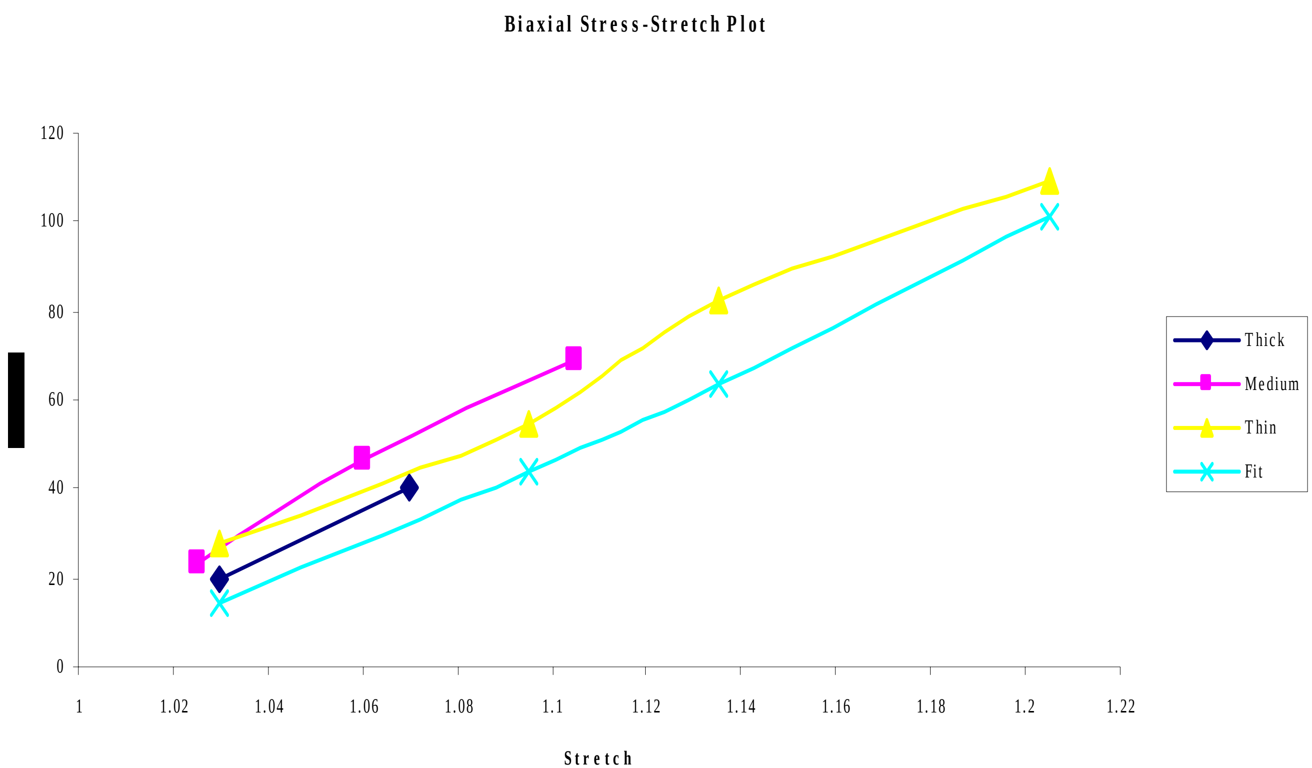
$$P_3 = \frac{\partial W}{\partial \lambda_3} = 0$$

Hence

$$\lambda_3 = \lambda^{-\frac{2\alpha}{\alpha+\beta}}$$

$$P = \mu\left(\lambda^{2\alpha\beta-\alpha-\beta} - \lambda^{-(\beta+1)}\right)$$

Testing Potential for Biaxial Loading Results



Conclusions

- The 3-D Hyperelastic potential is very accurate for both medium and thick sheets under uniaxial loading
- There are some discrepancies in the experimental measurements for biaxial test due to some practical difficulties
- The 2-D potential fits very accurately the data for thin sheet, hence it can be considered reliable for sheets with thickness less than 0.18 mm