Division of Engineering
Brown University

EN202: Math for Engineers and Physicists

Midterm Exam
Thursday March 19, 2009

NAME: ____

General Instructions

- No collaboration of any kind is permitted.
- You may consult your own lecture notes and homework during the course of this examination, but no other material. **XEROXED OR PRINTED NOTES AND HW SOLUTIONS ARE NOT PERMITTED**
- Write all your solutions in the space provided. No sheets should be added to the exam.
- If you find you are unable to complete part of a question, proceed to the next part. Explain as best you can how you would proceed, had you been able to answer the first part.
- Good luck!

Please sign the statement below

By affixing my name to this paper, I affirm that I have executed the examination in accordance with the Academic Honor Code of Brown University.

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1. (32 points) ____________

2. (30 points) ____________

3. (38 points) ____________

Extra Credit…. What is he saying?

TOTAL (100 points) ____________

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1. (32 points) Find the solution $u(x,y)$ to the initial value problem:

$$xyu_x + u_y = y, \;\; y \geq 0, \; (-\infty < x < \infty), \; u(x,0) = u_0(x).$$

The answer will involve the function u_0. Be sure to write the solution in the form $u(x,y)$.

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2. (30 points) A simple model for the propagation of wind-driven water waves in a channel is given below.

$$(2+u)u_x + u_y = 0, \ (-\infty < x < \infty, \ y \geq 0).$$

For the initial condition $u(x,0) = \sin(x)$, find the value of y for which the waves begin to break (vertical tangents appear). What are the positions of the vertical tangents?

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3. (38 points) Here is another equation:

$$u_{xx} - u_{yy} + u_x - u_y = 0, \ (y \geq 0, -\infty \leq x \leq \infty). \qquad (*)$$

a. Find expressions for the characteristic coordinates ξ and η.
b. Find an equation satisfied by the function $\bar{u}(\xi, \eta)$.
c. Solve the equation found above. You will find that the answer involves 2 undetermined functions, $f(\xi)$ and $g(\eta)$.
d. The solution to the PDE may be written as

$$u(x, y) = e^{-y}\phi(x - y) + \psi(x + y),$$

where ϕ and ψ are scalar functions of their arguments.

- If you got a solution in part c, reconcile your solution with the representation above.
- If you did not get a solution in part c, show by substitution that *u* given above satisfies the PDE $(*)$.

e. Determine the functions ϕ and ψ so that the solution satisfies the initial conditions $u(x, 0) = e^x$, $u_y(x, 0) = 0$, $(-\infty \leq x \leq \infty)$. Thus, find a solution $u(x,y)$ to this initial value problem.

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