



ENGN2020: Graduate Math for Engineers and Physicists

Exam II
Tuesday May 5, 2009

Division of Engineering
Brown University
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NAME: _____

General Instructions

- Your own handwritten notes, midterm exam and homework solutions are allowed. You can also refer to the midterm and homework problem statements, but no other printed or Xeroxed material is permitted.
- Write all your solutions on the exam.
- You may use any result from class notes, with proper citation.
- If you find you are unable to complete part of a question, proceed to the next part. Explain as best you can how you would proceed, had you been able to answer the first part.
- Good luck!

Please sign the statement below

By affixing my name to this paper, I affirm that I have executed the examination in accordance with the Academic Honor Code of Brown University.



1. _____ /25

2. _____ /12

3. _____ /13

4. _____ /25

5. _____ /25

TOTAL

_____ /100

1. (25 points) Look for a similarity solution of the form $u(x, y) = y^\alpha f(\eta)$, $\eta = x/y^\beta$ to the PDE:

$$u_{xx} - xu_y = 0, (x > 0, y > 0)$$

- a. Determine conditions on α and β and obtain an ODE for f .
- b. Find the solution to the PDE with the conditions
 $u(0, y) = 1$ ($y > 0$) and $u(x, 0) = 0$ ($x > 0$)

Problem 1, Continued

2. (12 points) Consider the wave equation on the half line with

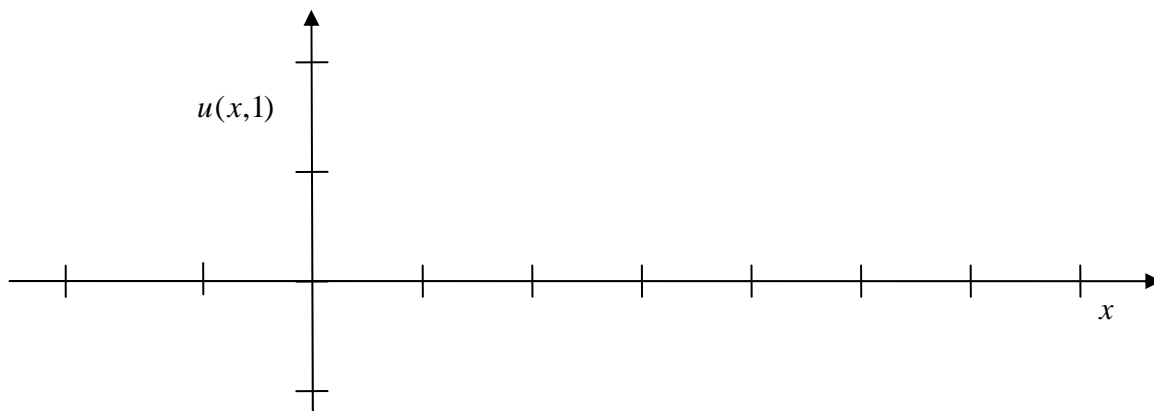
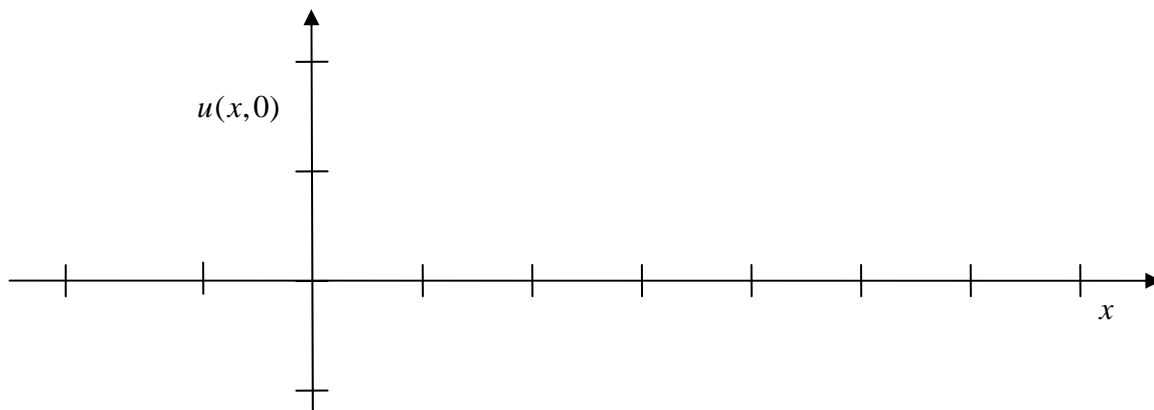
$$u_{xx} - u_{yy} = 0 \quad (0 < x < \infty, y > 0),$$

$$u(x, 0) = \begin{cases} \sin\left(\frac{\pi}{2}(x-1)\right) & 1 < x < 3 \\ 0 & \text{otherwise} \end{cases},$$

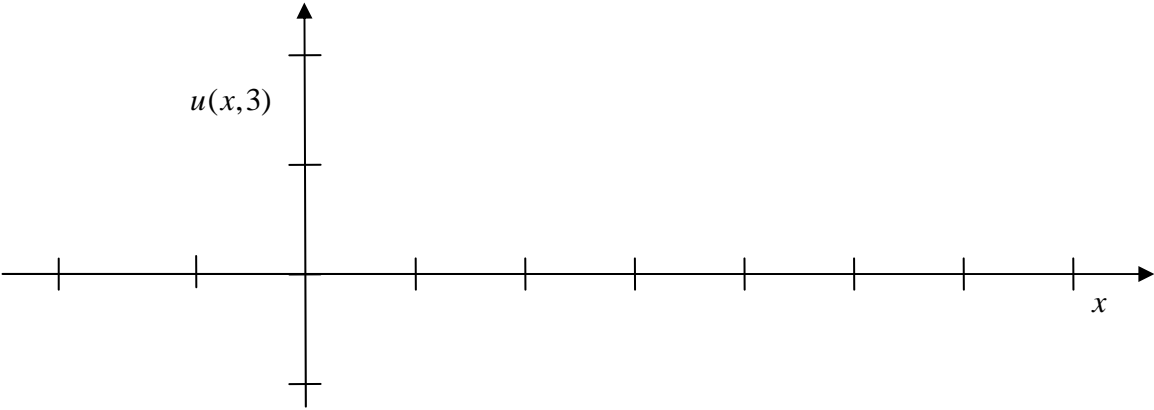
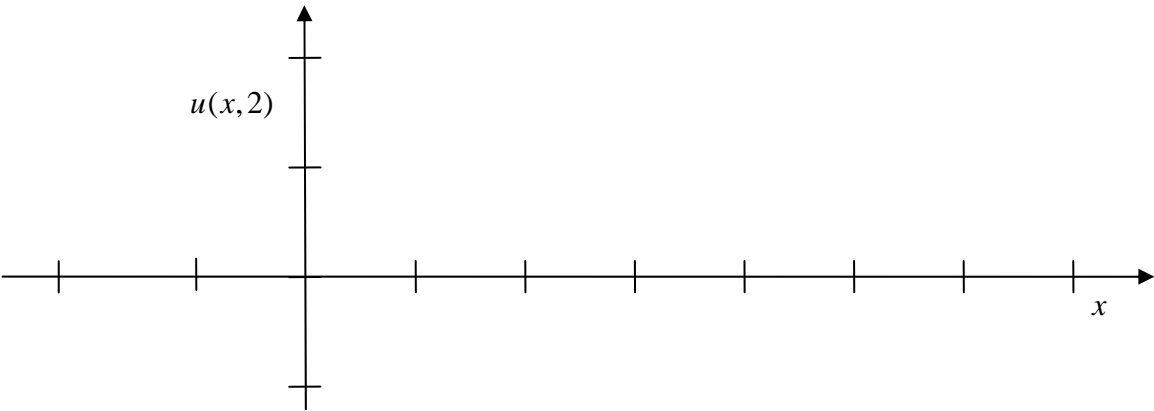
$$u_y(x, 0) = 0, \quad (x > 0)$$

$$u(0, y) = 0, \quad (y > 0)$$

On the graphs below and on the next page, sketch of the solution at $y=0, 1, 2,$ and 3 . Label the tick marks.



Problem 2, Continued



3. (13 points) Longitudinal waves in a large circular ring containing a small spring insert satisfy the following problem:

$$u_{xx} - u_{yy} = 0, \quad y \geq 0, \quad 0 < x \leq 1, \quad K(u(0, y) - u(1, y)) = u_x(0, y) = u_x(1, y), \quad (y \geq 0).$$

K is the spring constant. The solution to this problem is of the form

$$u(x, y) = \sum_{n=0}^{\infty} X_n(x) [A_n \cos \omega_n y + B_n \sin \omega_n y] \quad y \geq 0, \quad 0 < x \leq 1$$

Are the eigenfunctions X_n orthogonal? Why or why not?

Problem 3, Continued

4. (25 points) Use the Fourier Transform to find the solution to the following IVP:

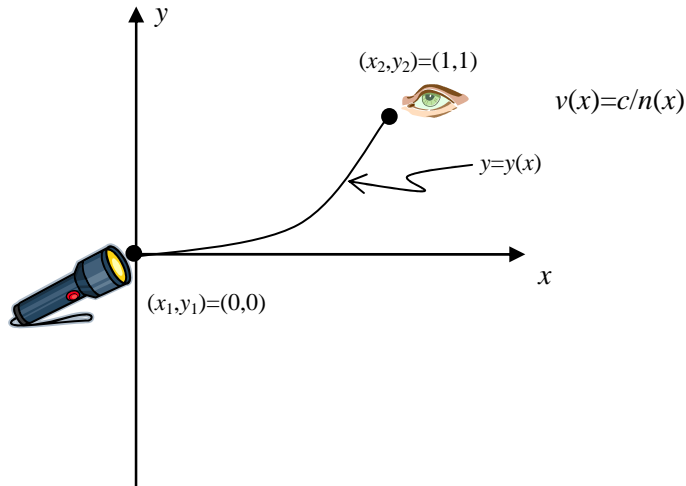
$$u_{xx} - u_y = \delta(x - y) \quad (-\infty < x < \infty, y > 0)$$

$$u(x, 0) = 0, \quad (-\infty < x < \infty)$$

You may leave the answer in the form of a single integral.

4. Problem 4, continued.

5. (25 points) According to Fermat's principle in optics, a ray of light traveling through a medium with variable refractive $n(x)$ index follows the path $y(x)$ for which its total transit time is a minimum. Recall that the speed v of light through a given medium is the speed c of light in a vacuum divided by refractive index n : $v=c/n$.



a. Show that the total transit time for the ray of light along the path from (x_1, y_1) to (x_2, y_2)

is
$$I[y] = \frac{1}{c} \int_{x_1}^{x_2} n(x) \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$$

b. If $n(x) = 1/x$ and $y(0) = 0, y(1) = 1$, find the path of light in the medium.

Problem 5, Continued

BONUS:

Three types of equations you've learned
During this long, arduous spring term.
Elliptic parabolic,
and then hyperbolic
Describe them for 5 bonus points earned.

Needless to say, you should rhyme
Even if it takes extra time!