Engineering 31: Mechanics of Solids and Structures

Laboratory Instruction Sheet

Measurement of Strain in an Aluminum Beverage Can

Objective:

To measure the strain in a pressurized aluminum can, to infer the corresponding state of stress, and to compare results to theoretical predictions.

1 Introduction

This experiment is intended to illustrate several important concepts. Firstly, you will measure the distribution of strain in a can using foil strain gages. This technique is by far the most commonly used method for strain measurement, and many structures and components are routinely instrumented using strain gages to monitor their performance during service. Secondly, you will use the strain gage measurements, together with some knowledge of the fundamental properties of stress and strain in a deformable solid, to deduce the state of stress in the can and compare your results to analytical predictions based on some simplifying assumptions. You will also measure the stress and strain in parts of the can where the geometry prohibits an analytical estimate. Finally, you will learn some practical details about aluminum cans, their design and mechanical properties.

2 Background

2.1 Strain measurements

In the present experiment, strain in the can is measured by means of electrical resistance foil strain gages. A strain gage is essentially a thin, continuous piece of wire or foil which is glued to the surface of a component. A typical configuration for a foil gage is illustrated in Figure 1. If the material to which the gage is attached stretches in the direction of the gage, then the gage also increases its length. As a result, the electrical resistance of the wire increases. By measuring this change in resistance, one can deduce the change in length of the wire, and so find the strain ϵ_n in the direction of the wire. The extensional

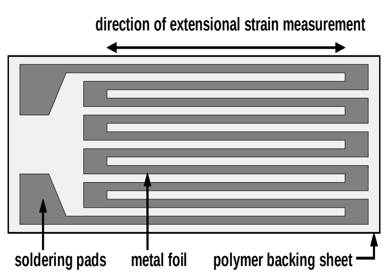

Figure 1: Schematic of a strain gage.

strain ϵ_n is related to the change in resistance of the wire ΔR by

$$\epsilon_n = \frac{1}{F}\frac{\Delta R}{R} \tag{1}$$

where R is the resistance of the unstrained gage (typically $120\,\Omega$), and F is a *gage factor* which is constant for a given gage. Values of F and R are provided by the manufacturer of the strain gage.

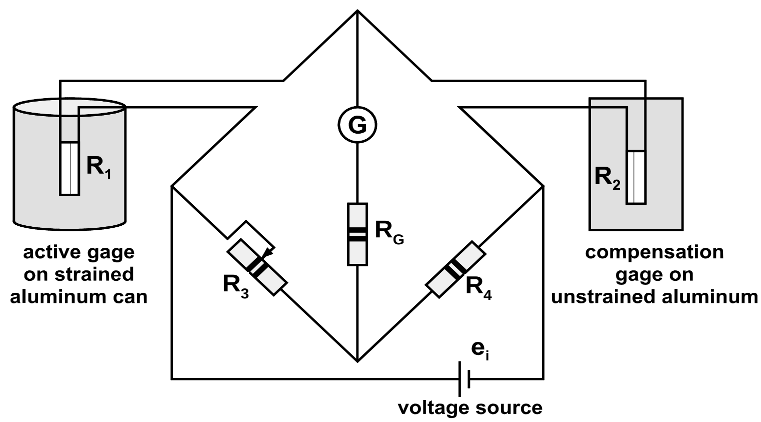

Figure 2: Schematic of the Wheatstone bridge circuit.

If one wishes to measure three components of strain ϵ_x , ϵ_y , and γ_{xy} at a point on the surface of a component, one must use at least three gages with different orientations attached to the same "point". An assembly of three gages is called a *strain gage rosette*.

To get an estimate of the resistance changes which must be measured, suppose that the commercial equipment can detect strains as small as $\epsilon_n = 10^{-6}$. Then, for $R = 120\,\Omega$ and $F = 2$ we find that $\Delta R = \epsilon_a F R = 10^{-6} \times 2 \times 120\,\Omega$. An electrical circuit, known as the *Wheatstone bridge* and shown in Figure 2, is one method available for measuring strain of this magnitude. To use the circuit, one first adjusts the variable resistor R_3 until the galvanometer G (a voltage measuring device) records zero voltage. This is known as *balancing* the bridge, and should be done with the specimen unstressed. Then, a load is applied to the specimen, the value of resistance R_1 changes and the voltage detected by the galvanometer is altered; the bridge becomes *unbalanced*. The voltage change detected by the galvanometer is roughly proportional to the strain experienced by the active gage.

The Wheatstone bridge is commonly used in null mode. In this mode, once the bridge becomes unbalanced due to change in resistance R_1, it is rebalanced by adjusting the resistance value of R_3. For a balanced bridge, the values of resistance are related by $R_3/R_4 = R_1/R_2$; any influence of temperature fluctuations in the laboratory are canceled out by the use of an unstrained compensation gage identical to the active gage. It follows that the change in resistance of R_1, say ΔR_1, is proportional to the change in resistance of R_3, say ΔR_3, necessary to rebalance the bridge according to

$$\Delta R_1 = \frac{R_2}{R_4}\Delta R_3 \tag{2}$$

The commercial equipment used in this experiment contains a voltage source e_i, the resistors R_3 and R_4 (where the magnitude of resistance of R_3 may be varied to balance the bridge), and a galvanometer to monitor the balance of the bridge circuit. There is also some circuitry to scale the galvanometer reading so as to correct for the effect of the gage factor F. The reading on the box displays microstrain directly, possibly scaled by multiples of 10.

2.2 Cans

A soda or beer can is a pressure vessel; the can Contents are pressurized in order to maintain carbonation. Hundreds of millions of these cans are produced daily in the US, and can design is a major research and development thrust at aluminum companies such as the Reynolds Aluminum Co. and ALCOA.

Can design is governed largely by economic considerations. Reynolds Metals makes 17 billion beverage cans a year which is 20% of the beverage can production capacity in the US. Reducing incoming metal thickness by 0.0001 inches (0.0025 mm) will reduce the metal cost by approximately $4m. Customer requirements are the beginning stage of can design. In this case, the customer is the beverage company. Modern filling and seaming equipment will apply an *axial* load to the can of approximately 200 lbs[1] during the filling operation. Since the manufacturing operation that reduces the diameter of the open end of the can (neck formation) applies 275 lbs of axial load, any can that can be necked will survive the filling process. An empty can should still accommodate an axial load of 250 lbs without buckling (for warehouse stacking), provided the load is very carefully applied. Another customer requirement is internal pressure resistance. For pasteurized beer, the can may be subject to pressures as high as 90 psi[2] during filling, packing and delivery. This means that the inverted dome at the bottom of the can must have a reversal pressure greater than 90 psi. Sodas at 100 degrees Fahrenheit (or 38 degrees Celsius) will also pressurize a can to 90 psi. This can easily occur when a can is left in the sun on a hot day. The domed bottom of the can is resistant to bulging under normal service pressures *below* 90 psi internal pressures and prevents catastrophic can failure under over-pressures. At a critical can pressure of roughly 100 psi, the can bottom will *snap-through*, that is, the originally inward or inverted dome will pop outward. This significantly increases the can volume, and in turn reduces the internal pressure of the gas and liquid within. Once the can bottom is reversed, however, continued increase in internal pressure may result in the explosive unraveling of the can lip, or a breakage at the scored pop-top opening. The dome is not quite spherical, because in addition to resisting static internal pressure, the can must survive being dropped while the filled can is being transported. A spherical dome has excellent pressure resistance but poor drop performance. Most can domes are now bi-radial, meaning that the dome has a relatively large radius in the center with a smaller radius at the edge for drop performance. In addition, the smaller the footprint diameter of the can, the more internal pressure it can withstand for a given metal thickness.

Cans must stack readily on top of each other. The outside geometry on the base of the can is designed to fit in a can top. The top is the thickest part of the can. The relatively large top thickness is required to keep this flat disk from bowing upward. In order to save metal, the radius of this disk is reduced, and the top of the can necks inward.

One might expect the best can design to have small end and bottom diameters, and to be made of very thin material. However, the economics of the manufacturing process dictate otherwise. Can makers must convert relatively thin flat sheet stock into very complex shapes by means of metal forming. The forming processes must be very efficient. Typical can plants operate at a line speed of about 1800 cans per minute.

Acknowledgement

This laboratory was developed by Professor Blume and Mr. Paul Rush. Professor Blume is grateful to Dr. Don Haulsee, Head Can Design Engineer at Reynolds Metals, for helpful discussions and information.

[1] 1 lb = 4.448 N

[2] 1 psi = 6.895 N/m^2

2.3 Pressure Vessels

A beverage can is a pressure vessel which, at least at the midsection, should be mechanically equivalent to a cylindrical pressure vessel. As shown in class, the nonzero stress components (or, more precisely, *non-negligible* stress components) for a cylindrical pressure vessel are

$$\sigma_\theta = p\frac{r}{t}\,, \quad \sigma_z = p\frac{r}{2t} \tag{3}$$

in which p is the internal pressure, r is the can radius, and t is the can wall thickness on the cylindrical sides. For isotropic elastic response, the nonzero strain components are

$$\epsilon_\theta = \frac{p(2-\nu)}{2E}\frac{r}{t}\,, \quad \epsilon_z = \frac{p(1-2\nu)}{2E}\frac{r}{t}\,, \quad \epsilon_r = -\frac{3p\nu}{2E}\frac{r}{t} \tag{4}$$

For aluminum the elastic modulus is $E = 70\,\mathrm{GPa}$ and the Poisson ratio is $\nu = 0.31$.

3 The experiment

3.1 Equipment and specimen

- Oil pump and pressure gage
- Strain indicator (essentially, a Wheatstone bridge circuit as discussed in sect. 2.1)
- A switching unit which allows the strain indicator to be connected to several gages in turn.
- Can instrumented with strain gages as shown in Figure 3.
- Instron machine
- Various calipers and measurement devices

3.2 Experimental Procedure

- Each member of the group will be given a *full* soda can. Open the can, drink the soda. Then measure all pertinent exterior dimensions of the can: length, circumference, diameter of the top, diameter of the middle, radius of the indented bottom dome, location of upper neck, etc. Cut the can open, and measure the wall thickness at various locations throughout the can. You may divide the work among group members, but each measurement should be taken by at least two members.
- The instrumented can (see Figure 3) is fitted with a pressurizing cap. The pump sends oil into the can. Pressurize the can and record the strain gage readings and the diameter and length of the can for internal pressures of 0, 20, 40, 60, and 80 psi. **Do NOT increase the pressure above 80 psi.**
- Remove the pressure and check the reading for each gage, and record the can length and circumference.
- Place an empty can in a universal testing machine with the help of a TA, and run compression tests. Allow the cans to buckle, and note the buckling load. Try this for three different cans. Pick cans for the area as pristine as possible.

3.3 Data reduction and report preparation

- Determine the complete state of strain at uniaxial gage locations 1 and 2, and at the 45° degree rosette gage (3, 4, and 5). At the uniaxial gages, you must incorporate what you know about the state of stress to determine the strain, because one measured value of strain is not enough by itself to yield the complete state of strain. Are the states of strain the same at locations 1 and 2? Compare the circumferential or *hoop* strain and axial or *longitudinal* strain at a point. Compare the circumferential strain ϵ_θ and the axial strain ϵ_z as determined from the diameter and length change measurements measurement with the recorded strain gage readings at gage 1. Explain any differences. Compare the measured strains with theory wherever possible. Show all calculations.

- Calculate the principal stresses from the measured strains at all locations. Compare the stresses with theory wherever possible. Try to account for any differences. At which location is the largest tensile stress found?

- Plot the largest principal stress versus p at location 1.

- The relationship $(2-\nu)/(1-2\nu) = \epsilon_\theta/\epsilon_z$ or $\nu = [(\epsilon_\theta/\epsilon_z) - 2]/[2(\epsilon_\theta/\epsilon_z) - 1]$ can be derived from equation (4). Evaluate Poisson's ratio from your experimental data and compare it with the value of 0.31 given in the literature. For ϵ_θ and ϵ_z, use incremental values at gage 1 and gage 2 respectively. Indicate why use of incremental values gives better agreement than use of net changes.

In addition to addressing the issues mentioned above, your report of the experiment should include the following sections:

- A cover page with title of project, your name, the names of all members of your lab group, the date of your lab session, and the date the report was turned in.

- A brief Introduction (no more than 1 page) describing the objective of the experiment, outlining the procedure that you used, and showing a sketch of the specimen with dimensions and locations of all strain gages.

- A section describing *briefly* your data reduction; a spreadsheet would be useful to do data reduction. This should include a description of the method you used to deduce the state of strain and principal stresses at gage location 1 and at the rosette gage (3, 4, and 5). State your assumptions, justify them briefly in a sentence or two, list the equations that you used and provide a few sample calculations. Refer to class notes or the textbook for details about how to do the calculations.

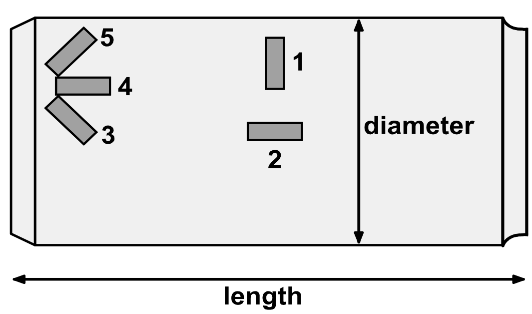

Figure 3: Instrumented soda can.