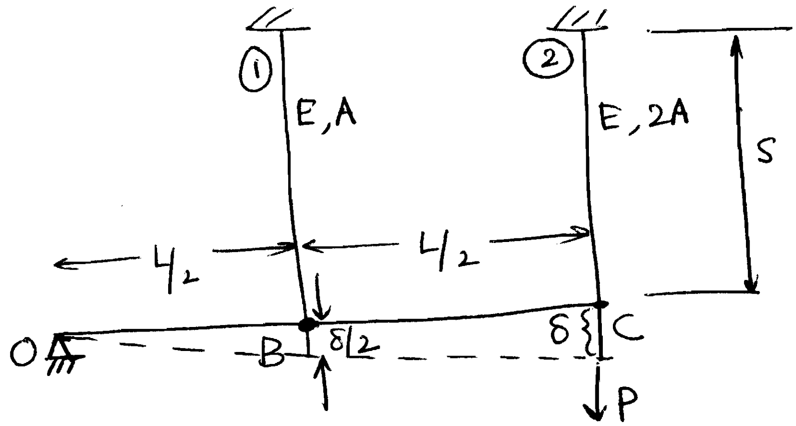
(1)

(a)

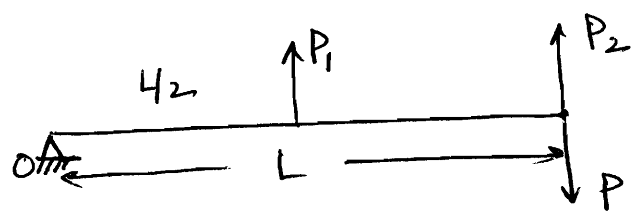


Strain in bar (1) $= \epsilon_1 = \dfrac{\delta/2}{S} = \dfrac{\delta}{2S}$

$\Rightarrow \sigma_1 = E\epsilon_1 = \dfrac{E\delta}{2S} \Rightarrow \boxed{P_1 = A\sigma_1 = \dfrac{EA\delta}{2S}}$ — (1)

strain in bar (2) $= \epsilon_2 = \dfrac{\delta}{S} \Rightarrow \sigma_2 = \dfrac{E\delta}{S} \Rightarrow \boxed{P_2 = \dfrac{2AE\delta}{S}}$ — (2)

From (1) & (2), $\boxed{P_2 = 4P_1}$ — (3)



Moment balance about O : $P_2 \cdot L + P_1 \cdot \frac{L}{2} = PL$

Using (3), $4P_1\cancel{L} + P_1\frac{\cancel{L}}{2} = P\cancel{L} \Rightarrow \frac{9}{2}P_1 = P \Rightarrow \boxed{P_1 = \frac{2}{9}P}$ — (4)

$\Rightarrow P_2 = 4P_1 \Rightarrow \boxed{P_2 = \frac{8}{9}P}$ — (5)

From (1), $\delta = \dfrac{2SP_1}{EA} = \dfrac{2S}{EA} \cdot \dfrac{2}{9}P = \dfrac{4PS}{9EA}$

$\Rightarrow \boxed{\delta = \dfrac{4}{9}\dfrac{PS}{EA}}$ — (6)

(b) Let the force in bar (1) be P_1 & that in bar (2) be P_2.

Strain in bar (1) and strain in bar (2)

$$\epsilon_1 = \frac{P_1}{AE} + \alpha \Delta T, \quad \text{and} \quad \epsilon_2 = \frac{P_2}{2AE} + \alpha \Delta T$$

Let the deflection of point C due to combined action of P and ΔT be d (downward). Then the deflection of B is $\frac{d}{2}$.

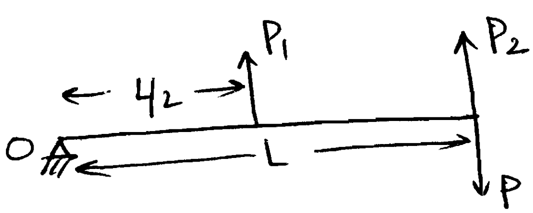
$$\Rightarrow \epsilon_1 = \frac{d}{2S} = \frac{P_1}{AE} + \alpha \Delta T \Rightarrow \frac{d}{S} = \frac{2P_1}{AE} + 2\alpha \Delta T \longrightarrow (7)$$

$$\& \quad \epsilon_2 = \frac{d}{S} = \frac{P_2}{2AE} + \alpha \Delta T \longrightarrow (8)$$

From (7) & (8)

$$\frac{2P_1}{AE} + \cancel{2}\alpha \Delta T = \frac{P_2}{2AE} + \cancel{\alpha \Delta T}$$

$$\Rightarrow \boxed{P_2 = 4P_1 + 2AE\alpha\Delta T} \text{ —— (9)}$$



Moment balance about O.

$$P_2 \cdot \cancel{L} + P_1 \cdot \frac{\cancel{L}}{2} = P\cancel{L}$$

Using (9)

$$\Rightarrow 4P_1 + 2AE\alpha\Delta T + \frac{P_1}{2} = P \Rightarrow \frac{9}{2}P_1 = P - 2AE\alpha\Delta T.$$

$$\Rightarrow \boxed{P_1 = \frac{2}{9}P - \frac{4}{9}AE\alpha\Delta T} \text{ —— (10)}$$

$$\therefore P_2 = \frac{8}{9}P - \frac{16}{9}AE\alpha\Delta T + 2AE\alpha\Delta T = \frac{8}{9}P + \frac{2}{9}AE\alpha\Delta T$$

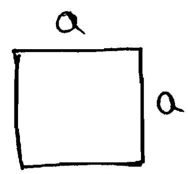
$$\Rightarrow \boxed{P_2 = \frac{8}{9}P + \frac{2}{9}AE\alpha\Delta T} \qquad \text{———— (11)}$$

(c) From (10), $P_1 = 0$ when

$$\frac{2}{\cancel{9}}P = \frac{4}{\cancel{9}}AE\alpha\Delta T$$

$$\Rightarrow \boxed{\Delta T = \frac{P}{2AE\alpha}} \qquad \text{———— (12)}$$

②



$$\sigma_y = \frac{M\left(\frac{a}{2}\right)}{I} = \frac{Ma}{2\cdot\frac{a^4}{12}} = \frac{6M}{a^3}.$$

$$I = \frac{a^4}{12} \qquad \Rightarrow a^3 = \frac{6M}{\sigma_y}.$$

(i) For Aluminum alloy

$$a_{al}^3 = \frac{6M}{\sigma_y} \Rightarrow a_{al} = \left(\frac{6M}{\sigma_y}\right)^{1/3}$$

$$\underset{(M_{al})}{\text{Mass}} = d\,L\,a_{al}^2 = dL\left(\frac{6M}{\sigma_y}\right)^{2/3}.$$

$$a_{st} = \left(\frac{6M}{3\sigma_y}\right)^{1/3}$$

$$M_{st} = (3d)L\cdot\left(\frac{6M}{3\sigma_y}\right)^{2/3} = 3dL\left(\frac{6M}{3\sigma_y}\right)^{2/3}.$$

$$\Rightarrow \frac{M_{al}}{M_{st}} = \frac{\cancel{d}L\left(\frac{\cancel{6M}}{\cancel{\sigma_y}}\right)^{2/3}}{3\cancel{d}L\left(\frac{\cancel{6M}}{3\cancel{\sigma_y}}\right)^{2/3}} = \frac{1}{3\cdot\frac{1}{3^{2/3}}} = \frac{1}{3^{1/3}} < 1.$$

$\Rightarrow$ Aluminum beam is lighter.

(ii) $\theta = \frac{ML}{EI} = \frac{12\,ML}{E \cdot a^4} \Rightarrow a^4 = \frac{12ML}{E\theta}$.

$$\Rightarrow a_{al} = \left(\frac{12ML}{E\theta}\right)^{1/4} \quad \& \quad a_{st} = \left(\frac{12\,ML}{3\,E\theta}\right)^{1/4}.$$

$$M_{al} = dL\, a_{al}^2 = dL \left(\frac{12\,ML}{E\theta}\right)^{1/2}.$$

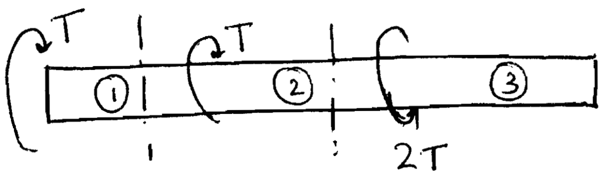
$$M_{st} = 3\,dL \left(\frac{12\,ML}{3\,E\theta}\right)^{1/2}.$$

$$\Rightarrow \frac{M_{al}}{M_{st}} = \frac{dL\left(\frac{12\,ML}{E\theta}\right)^{1/2}}{3\,dL\left(\frac{12\,ML}{3\,E\theta}\right)^{1/2}} = \frac{1}{3 \cdot \frac{1}{3^{1/2}}} = \frac{1}{3^{1/2}} < 1.$$

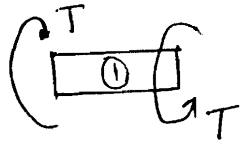
$\Rightarrow$ Aluminum beam is lighter!

(3)

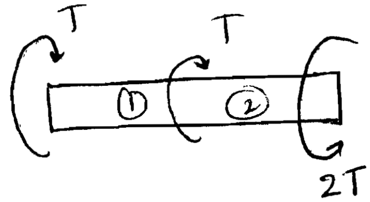
(a)



$J_1 = J_2 = J_3 = \frac{\pi R^4}{2} = J$



$$\Rightarrow \phi\Big|_{at\ x=\frac{L}{3}} = \frac{T(L/3)}{GJ} \text{ (c.c.w)} = \boxed{\frac{TL}{3GJ} \text{ (c.c.w)}} = \frac{2TL}{3\pi G R^4}$$



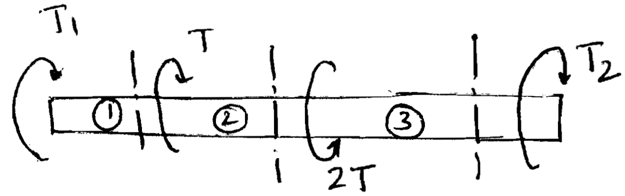
$$\text{In } 2,\quad \phi_2 = \frac{2T\left(\frac{L}{3}\right)}{GJ} = \frac{2TL}{3GJ} \text{ (c.c.w)}.$$

$$\Rightarrow \phi\Big|_{x=\frac{2L}{3}} = \frac{TL}{3GJ} + \frac{2TL}{3GJ} = \frac{TL}{GJ} = \boxed{\frac{2TL}{\pi G R^4} \text{ (c.c.w)}}$$

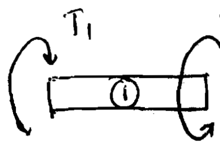
Since there is no torque in 3, $\phi_3 = 0$.

$$\Rightarrow \phi\Big|_{x=L} = \phi\Big|_{x=\frac{2L}{3}} = \frac{2TL}{\pi G R^4} \text{ (c.c.w)}.$$

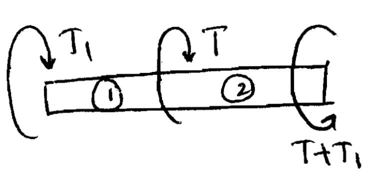
(b) Free body diagram.



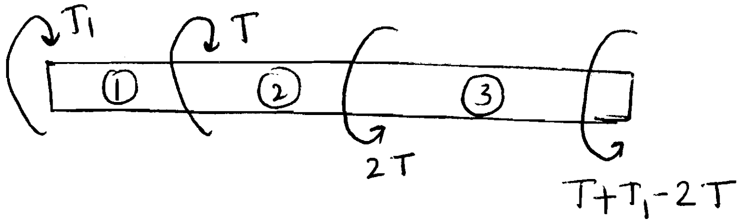
$$\boxed{T_1 + T + T_2 = 2T}$$



$$\Rightarrow \phi_1 = \frac{T_1 L}{3GJ} \text{ (c.c.w)}$$



$$\Rightarrow \phi_2 = \frac{(T+T_1)L}{3GJ} \text{ (c.c.w)}$$



$$\Rightarrow \phi_3 = \frac{(T+T_1-2T)L}{3GJ} \ (c.c.w).$$

$$\phi_1 + \phi_2 + \phi_3 = 0$$

$$\Rightarrow \frac{T_1 L}{3GJ} + \frac{(T+T_1)L}{3GJ} + \frac{(T+T_1-2T)L}{3GJ} = 0$$

$$\Rightarrow 3T_1 = 0 \Rightarrow \boxed{T_1 = 0}$$

$$\Rightarrow \boxed{T_2 = T}$$

$$\Rightarrow \boxed{\begin{aligned} \phi\Big|_{x=\frac{L}{3}} &= \frac{T_1 L}{3GJ} = 0 \\ \phi\Big|_{x=\frac{2L}{3}} &= \phi_1 + \phi_2 = \frac{TL}{3GJ} \ (c.c.w) \end{aligned}} \boxed{= \frac{2}{3\pi}\frac{TL}{GR^4} \ (c.c.w)}$$