



EN 31 Mechanics of Solids and Structures

Midterm Examination 2006

Name _____

General Instructions

- You may refer to your own lecture notes, homework solutions and any handouts distributed in class.
- Make diagrams and sketches as clear as possible, and show all your derivations clearly. Incomplete solutions will receive only partial credit, even if the answer is correct.
- **Ensure that what you write is legible and understandable to the grader.**



Problem 1 (40 points): _____

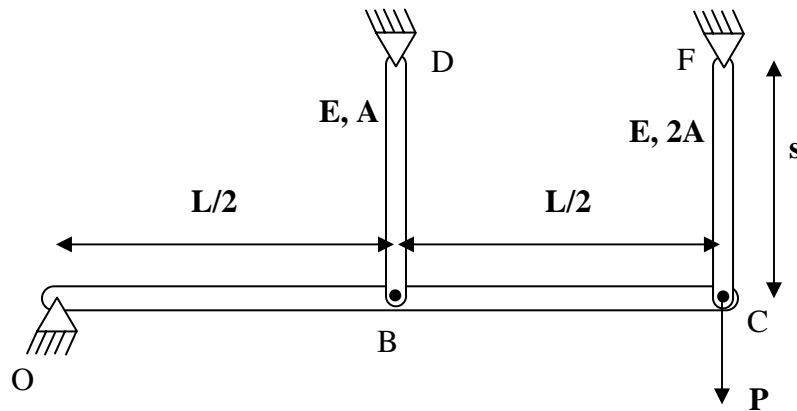
Problem 2 (30 points): _____

Problem 3 (30 points): _____

Total (100 points): _____

1. A rigid bar OBC of length L is supported at three points as shown. It is pinned at O and connected to two elastic bars BD and CF. BD and CF have pin joints at both ends, are made of a material with elastic modulus of E and are of length s . The area of cross section of BD is A and that of CF is $2A$. A force P is applied at C as shown.

- Determine the downward deflection δ of point C (in terms of P , E , A and s). What are the forces in BD and CF in terms of P ?
- The temperature of BD and CF is then increased by ΔT . The coefficient of thermal expansion of BD and CF is α . What are the forces in BD and CF due to the combined effect of the load P and the temperature increase ΔT (in terms of P , E , A , ΔT and α)? Note that the temperature increase has no effect on the rigid bar.
- At what temperature ΔT (in terms of P , E , A and α) does the force in BD become zero?



2. You are asked to design a beam of length L , with a square cross section of side a , to withstand a bending moment M . You are given a choice of two materials: an aluminum alloy and a steel. The properties of each material are given below.

Aluminum alloy

Young's modulus = E

Yield stress = σ_y

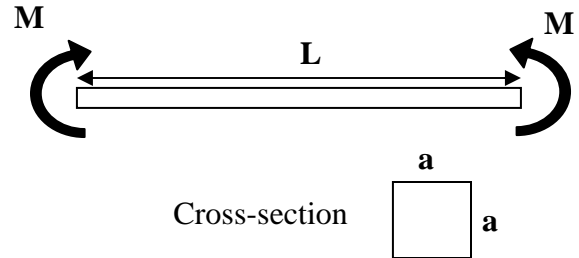
Mass density = d

Steel

Young's modulus = $3E$

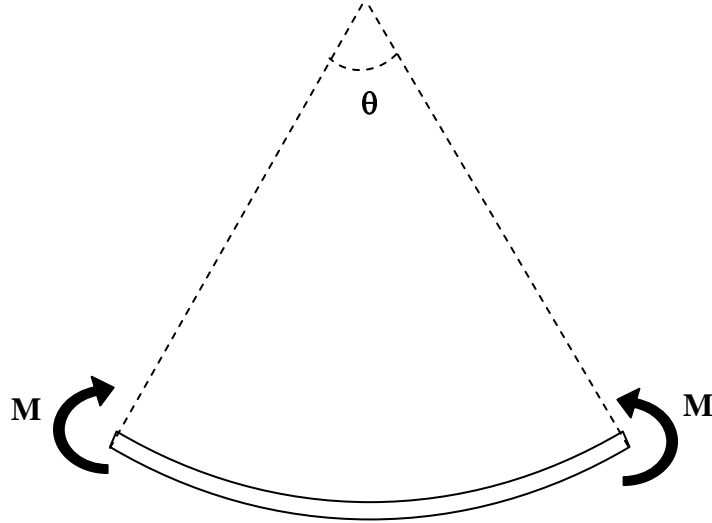
Yield stress = $3\sigma_y$

Mass density = $3d$



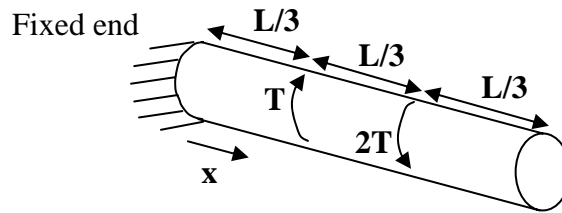
(i) Find the sizes a_{al} and a_{st} in terms of M and σ_y , if the beam is made of aluminum and steel respectively, such that the maximum stress in the beam is equal to the yield stress of the material. Find the mass of each bar (in terms of M , σ_y , L , d). Which beam is lighter, aluminum or steel?

(ii) Now, find the sizes a_{al} and a_{st} , if the beams made of aluminum and steel respectively, such that the maximum angle of rotation is equal to θ (in terms of M , E , L and θ). Find the mass of each beam (in terms of M , E , L , d , θ). Which beam is lighter, aluminum or steel? Assume that the beams do not yield and remain elastic throughout the deformation.



3. Consider a bar of solid circular cross section of radius R and of length L as shown below. The bar is made of a material with shear modulus G .

(a) The bar is subjected to torques T and $2T$ as shown at $x = L/3$ and $x = 2L/3$ respectively. What is the angle of twist at $x = L/3$, $x = 2L/3$ and $x = L$ (in terms of G , L , T and R)? Indicate the direction of the twist as well.



(b) What is the angle of twist (and the direction of the twist) at $x = L/3$ and $x = 2L/3$ if the other end is also fixed (in terms of G , L , T and R)?

