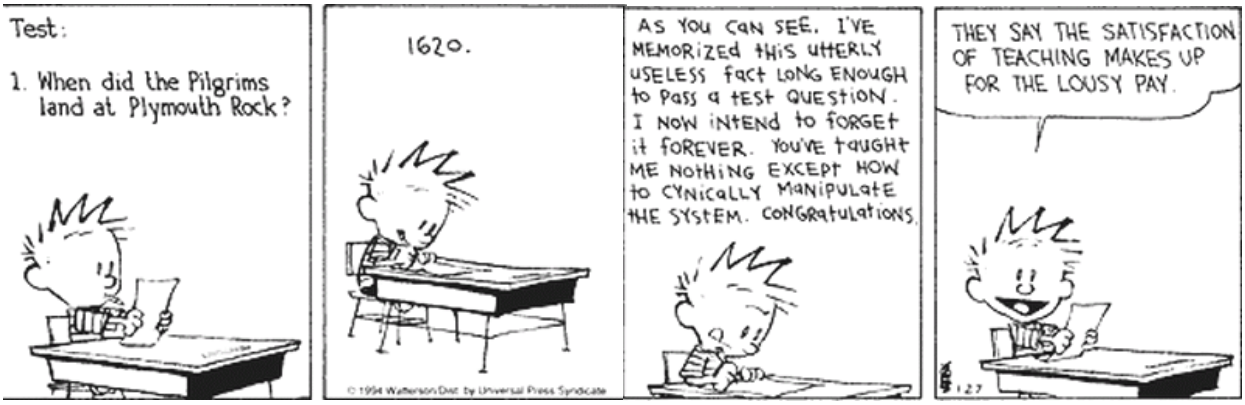
EN 31 Mechanics of Solids and Structures

Final Examination 2006

Name _____

General Instructions

- You may refer to your textbook, lecture notes, homework solutions and any material distributed in class.
- Make diagrams and sketches as clear as possible, and show all your derivations clearly. Incomplete solutions will receive only partial credit, even if the answer is correct.
- **Ensure that what you write is legible and understandable to the grader. The flow of your answer must be from the top of the page to the bottom. If you scramble your solution with no obvious flow, you will lose points, even if the answer is correct. If you turn the grader into a detective, searching for the flow of solution and logic, you will not receive full credit for your work.**
- **Do not try to fit your solution into the least possible area. Use the provided space generously and ask for additional blank pages if necessary.**



Problem 1 (25 points): _____

Problem 2 (25 points): _____

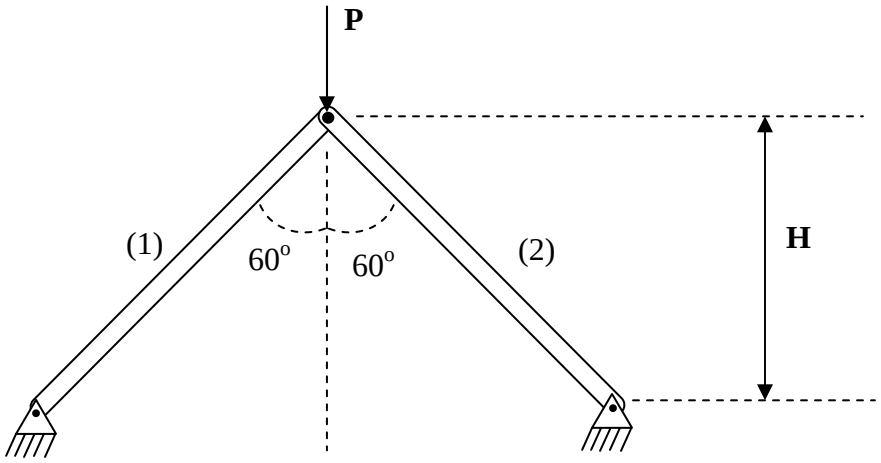
Problem 3 (25 points): _____

Problem 4 (25 points): _____

Total (100 points):

1. Consider the truss shown in the figure below, which is subjected to a force **P** as shown. Each rod of the truss is made of a material of elastic modulus **E**; has a circular cross section (radius of cross section **R**). All joints are pin joints.

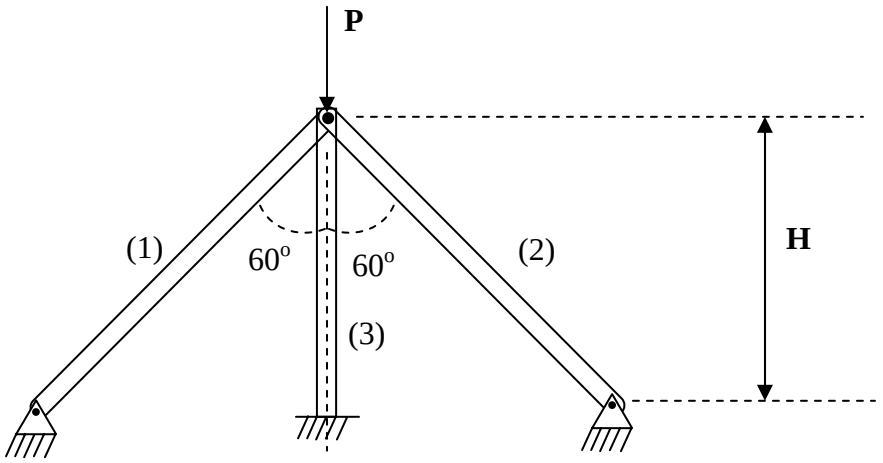
(a) What is the critical force **P** at which one or both rods buckle (in terms of **E**, **R**, **H**)?
(b) Sketch the buckled shape of the truss.



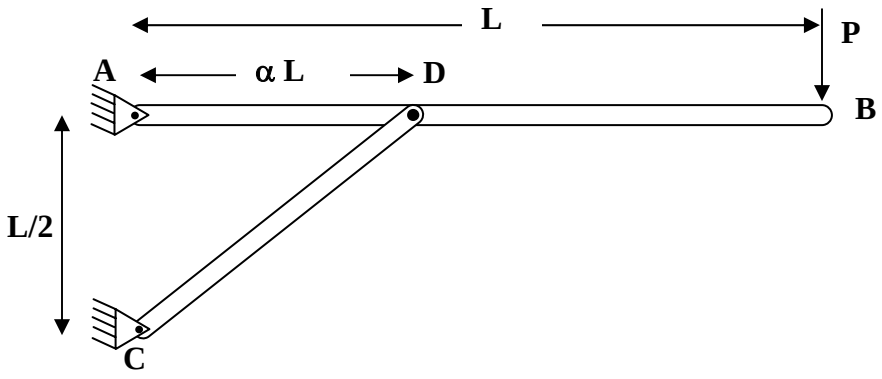
(c) A third rod of the same material, with a circular cross section of radius **R/2**, of length **H**, is added to the truss as shown below. The top end is connected to the other two rods with a pin joint, whereas, it has "fixed" boundary condition at the bottom end as shown.

(i) As **P** is increased, which rod(s) buckle(s) first and what is the corresponding critical buckling load $\mathbf{P_{cr}}$ (in terms of **E**, **R**, **H**)?

(ii) Sketch the buckled shape of the truss.



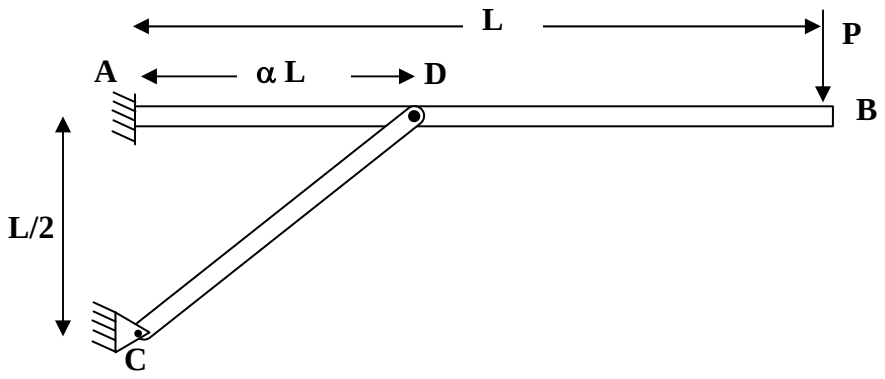
2. A beam **AB** (elastic modulus **E** and moment of inertia **I**) is supported as shown by another beam **CD** (elastic modulus **E** and moment of inertia **I**). The length of the beam **AB** is **L**; the distance **AD** is α **L**, where α is a constant whose value is between **0** and **1**. All joints are pin joints.



(a) What is the critical load $\mathbf{P_{cr}}$ at which **CD** buckles (in terms of **E, I, L,** α)?
(b) Where should the joint **D** be located to maximize the critical load $\mathbf{P_{cr}}$ (i.e. what is the value of α for which $\mathbf{P_{cr}}$ is maximum)?

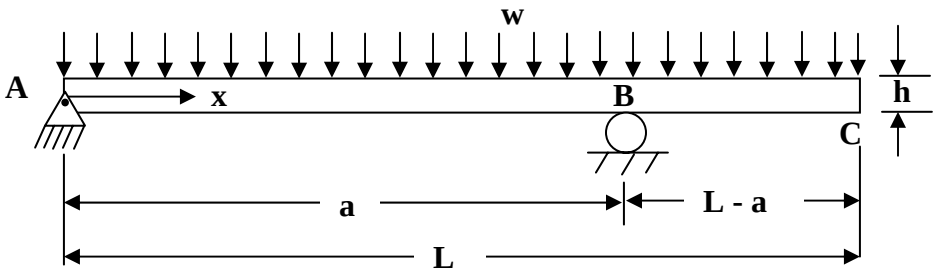
(Note: You can neglect the deflection of the point **D** due to axial compression in **CD**)

(c) The boundary condition at **A** is changed to "fixed" as shown in the figure below. For $\alpha = ½$, what is the critical load $\mathbf{P_{cr}}$ at which **CD** buckles (in terms of **E, I, L,** α)?



(Note: You can neglect the deflection of the point **D** due to axial compression in **CD**)

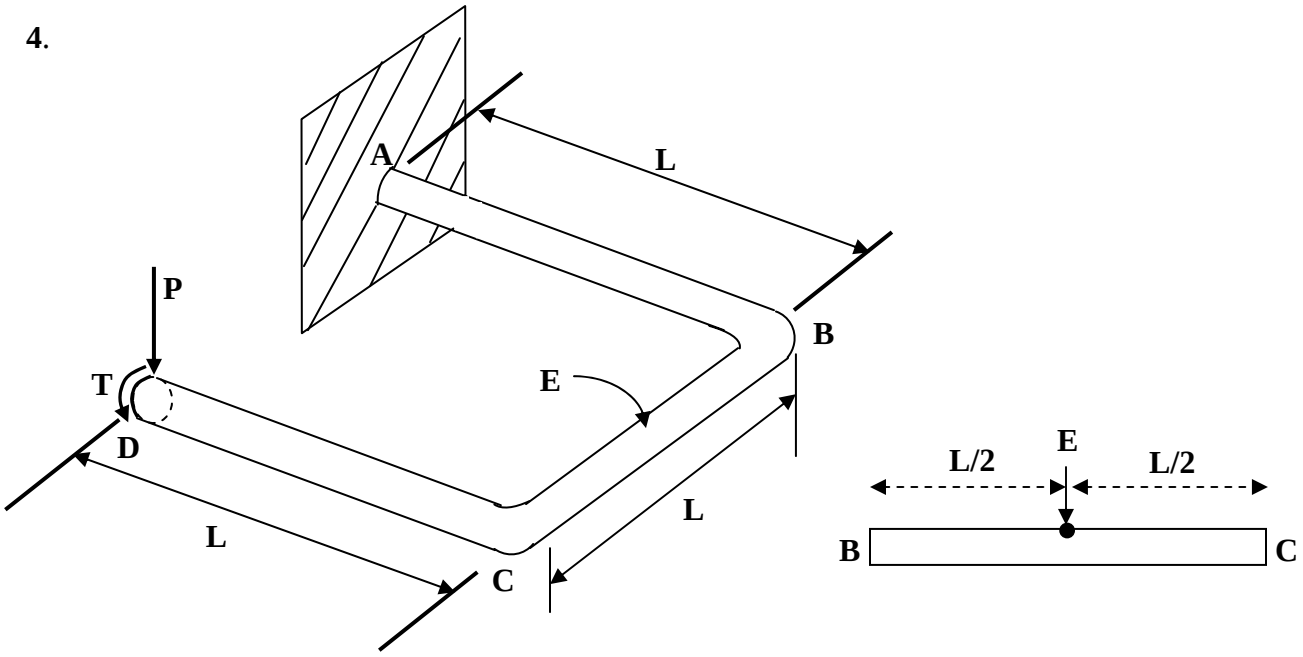
3. A beam is supported as shown (pin joint at **A** and a roller support at **B**). It is subjected to a uniformly distributed load **w** as shown. The thickness of the beam is **h**, Young's modulus **E** and moment of inertia **I**.



(a) By taking **a = 3L/4,** find expressions (in terms of **w, L** and **x**) for the shear force **V(x)** and the bending moment **M(x)** along the length of the beam and sketch them (shear force and bending moment diagrams). On the diagrams, show the values of the shear force and the bending moment (in terms of **w** and **L**) at **A, B, C** and any other discontinuities in the plot.

(b) It was determined that the bottom of the beam is very weak in tension and we need to adjust the value of **a** such that the entire bottom of the beam (**$y = -h/2,\ 0 \le x \le L$**) is under compression. Determine the range of values for **a** (in terms of **L**)**,** which will assure that the stress is compressive everywhere along the beam bottom.

4.



A U shaped bar of circular cross section (radius **a**) is fixed at one end and is subjected to a force **P** and a torque **T** at the other end (**D**) as shown. Note that the direction of **P** is perpendicular to the plane of the bar. The bar is made of a material of Young's modulus **E** and its Poisson's ratio $\nu = 0$.

a) What is the shear modulus **G** in terms of **E**? What is the polar moment of inertia of the cross section **J** in terms of the moment of inertia for bending **I**?
b) What is the stored elastic energy in each segment **AB, BC** and **CD** (in terms of **P, T, L** and the moment of inertia **I**)?
c) Suppose that the magnitude of **T** is equal to **PL** (product of **P** and **L**); using the Castigliano's theorem, determine the vertical deflection of point **D** in the direction of **P.** (express in terms of **P, L, a, E**).
d) If **T = PL**, what are the principal stresses at point **E**, which is the top most point of the bar, half-way between **B** and **C** (i.e. **CE =EB = L/2**) ? What is the maximum shear stress at **E**? (in terms of **P, L, a**).