Mechanics of robust and releasable adhesion in biology: Bottom–up designed hierarchical structures of gecko

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Abstract

Gecko and many insects have evolved specialized adhesive tissues with bottom–up designed (from nanoscale and up) hierarchical structures that allow them to maneuver on vertical walls and ceilings. The adhesion mechanisms of gecko must be robust enough to function on unknown rough surfaces and also easily releasable upon animal movement. How does nature design such macroscopic sized robust and releasable adhesion devices? How can an adhesion system designed for robust attachment simultaneously allow easy detachment? These questions have motivated the present investigation on mechanics of robust and releasable adhesion in biology. On the question of robust adhesion, we introduce a fractal gecko hairs model, which assumes self-similar fibrillar structures at multiple hierarchical levels mimicking gecko’s spatula ultrastructure, to show that structural hierarchy plays a key role in robust adhesion: it allows the work of adhesion to be exponentially enhanced with each added level of hierarchy. We demonstrate that, barring fiber fracture, the fractal gecko hairs can be designed from nanoscale and up to achieve flaw tolerant adhesion at any length scales. However, consideration of crack-like flaws in the hairs themselves results in an upper size limit for flaw tolerant design. On the question of releasable adhesion, we hypothesize that the asymmetrically aligned seta hairs of gecko form a strongly anisotropic material with adhesion strength strongly varying with the direction of pulling. We use analytical solutions to show that a strongly anisotropic elastic solid indeed exhibits a strongly anisotropic adhesion strength when sticking on a rough surface. Furthermore, we perform finite element calculations to show that the adhesion strength of a strongly anisotropic attachment pad exhibits essentially two levels of adhesion strength depending on the direction of pulling, resulting in an orientation-controlled switch between attachment and...
1. Introduction

Among hundreds of animal species for which adhesion plays an important role for survival, gecko stands out in terms of body weight and its extraordinary ability to maneuver on vertical walls and ceilings (Scherge and Gorb, 2001). Recent experimental measurements (Autumn et al., 2000, 2002; Autumn and Peattie, 2002; Huber et al., 2005) have provided evidence that the adhesion ability of gecko is primarily due to the van der Waals adhesion (Israelachvili, 1992) between the contact surfaces (e.g., walls or ceilings) and gecko’s feet which contain hundreds of thousands of keratinous hairs called setae (Figs. 1(a) and (b)); each seta is about 110 μm long and branches near its tip region into hundreds of thinner fibrils called spatulae arranged in a fractal-like hierarchical pattern (Fig. 1(c)). While it is remarkable that gecko can make use of the relatively weak van der Waals interactions to maneuver on unpredictable rough surfaces under harsh environmental conditions, it may be even more impressive that such robust adhesion appears to be easily releasable during animal locomotion. What are the mechanic principles behind such robust and releasable adhesion in biology?

Contact mechanics theories have been used to understand adhesion mechanisms in both engineering and biology. The classical Hertz theory (1882) assumes no adhesive interactions between contacting objects. Johnson et al. (1971) extended the Hertz theory to contact between adhesive elastic spheres and developed the JKR (Johnson–Kendall–Roberts) model in which the size of the contact area is determined via a balance between
elastic and surface energies similar to Griffith’s (1921) criterion for crack growth in an elastic solid. The JKR theory introduces into the Hertz solution an additional crack-like singular term which satisfies the Griffith condition near the contact edge. While the JKR theory is quite appropriate for modeling contact between large and soft materials, the assumption of a crack-like singular field becomes increasingly inaccurate for small and stiff materials, in which case different assumptions on the elastic deformation of contacting objects have led to the models of DMT (Derjaguin–Muller–Toporov) (Derjaguin et al., 1975) and Bradley (Bradley, 1932). Maugis (1992) generalized the Dugdale model of a crack in a plastic sheet (Dugdale, 1960) to adhesive contact and developed a more general model (Maugis–Dugdale model) that includes the JKR and DMT models as two limiting cases. More recent studies have further extended these theories to viscoelastic materials (Hui et al., 1998; Haiat et al., 2003), coupled normal and shear loads (Kim et al., 1998) and biological attachments (Arzt et al., 2002, 2003; Autumn et al., 2002; Persson, 2003a; Gao and Yao, 2004; Gao et al., 2005; Spolenak et al., 2005; Huber et al., 2005).

For contact between single asperities, one can define adhesion strength as the tensile force per unit contact area at pull-off, which can be maximized at the theoretical adhesion strength via size reduction (Persson, 2003b; Gao and Yao, 2004; Gao et al., 2005; Glassmaker et al., 2005). In this respect, it is interesting to note that the existing contact mechanics theories, including JKR, DMT and Maugis–Dugdale models, all predicted infinite adhesion strength as the size of contacting objects is reduced to zero. This behavior is unphysical because the adhesion strength cannot exceed the theoretical strength of adhesive interaction. The fact that this behavior also occurs in the Maugis–Dugdale model is especially peculiar since the original Dugdale model correctly predicted that the fracture strength is bounded by the yield strength of the material. Gao et al. (2005) found that the root of this unphysical behavior of the classical contact models stems from the original Hertz approximation of contact surfaces as parabolas, which is strictly valid only if the size of the contact area is much smaller than the overall dimension of the contacting objects; the lack of strength saturation in these models is thus explained from the fact that the parabolic approximation fails in the limit of very small contacting structures. As an example, Gao et al. (2005) showed that, if the exact geometry of a sphere in contact with a flat surface is considered, the adhesion strength indeed saturates at the theoretical strength as the diameter of the sphere is reduced to zero. On the other hand, Gao and Yao (2004) showed that the adhesion strength can in principle approach the theoretical strength for any contact size via shape optimization. In practice, interfacial crack-like flaws due to surface roughness or contaminants inevitably weaken the actual adhesion strength. Gao et al. (2005) performed finite element calculations to show that the adhesion strength of a flat-ended cylindrical punch in partial contact with a rigid substrate saturates at the theoretical strength below a critical radius around 200 nm for the van der Waals interaction. Similar discussions of strength saturation for small contacting objects have been made by Persson (2003b) for a rigid cylindrical punch on an elastic half-space and by Glassmaker et al. (2005) for an elastic cylindrical punch in perfect bonding with a rigid substrate. Gao and Yao (2004) showed that the theoretical strength can be achieved by either optimizing the shape of the contact surfaces or by reducing the size of the contact area; the smaller the size, the less important the shape. A shape-insensitive optimal adhesion can be realized below a critical contact size, which can be related to the intrinsic capability of a small-scale material to tolerate crack-like flaws (Gao et al., 2003, 2004; Gao and Chen, 2005). Hui et al. (2004) and Glassmaker et al. (2005) demonstrated that fibrillar
structures with slender elastic fibrils can significantly enhance the adhesion strength. Northen and Turner (2005) made use of massively parallel MEMS processing technology to produce hierarchical hairy adhesive materials containing single slender pillars coated with polymer nanorods, and reported significantly improved adhesion in such multiscale systems.

In contrast to the increasing volume of research on adhesion enhancement, the question of how adhesion is released upon animal movement has so far received relatively little attention. Autumn et al. (2000) reported experimental data that the pull-off force of an individual seta of gecko depends strongly on the pulling angle. Gao et al. (2005) numerically simulated the pull-off force of a single seta and found that the asymmetrical alignment of seta allows the pull-off force to vary strongly (more than an order of magnitude) with the direction of pulling.

Previous studies have provided significant insights into various aspects of adhesion mechanisms in biology. However, a general understanding is still lacking with respect to a number of critical issues. First, robust adhesion at the level of a single hair or fiber does not automatically address the problem of robust adhesion on rough surfaces at macroscopic scales. It has been shown that size reduction can result in optimal adhesion strength at the level of a single fiber (Gao and Yao, 2004; Gao et al., 2005; Glassmaker et al., 2005). However, it is not clear how this size-induced optimization might work at the system level of hierarchical structures. Similarly, releasable adhesion at the level of a single seta (Autumn et al., 2000; Gao et al., 2005) does not provide full explanations on how releasable adhesion is achieved in macroscopic contact. The present paper is aimed to address the basic mechanics principles which underline these issues. For robust adhesion, we show that the fractal-like spatula ultrastructure of gecko provides a systematical strategy to optimize adhesion strength at larger length scales, even in the presence of random interfacial crack-like flaws due to surface roughness. We show that, given sufficient hierarchical levels, a fractal hairy system can be designed using a bottom–up approach to achieve robust, flaw tolerant adhesion at any macroscopic length scales. However, consideration of crack-like flaws in the hairs themselves imposes an upper bound on the length scale for robust adhesion. For releasable adhesion, we show that macroscopic elastic anisotropy allows the adhesion strength to vary strongly with the direction of pulling, leading to an orientation-controlled switch between attachment and detachment. The bottom–up design principles of the hierarchical structures of gecko provide not only a foundation to understand more general adhesion mechanisms in biology but also suggest novel adhesive materials for engineering applications.

2. Bottom–up designed hierarchical structures for robust adhesion

2.1. Flaw tolerant adhesion of a single fiber

Adhesive contact between elastic objects usually fail by propagation of crack-like flaws initiated at poor contact regions around surface asperities, impurities, trapped contaminants, etc. As an external load is applied to pull the contacting objects apart, stress concentration is induced near the edges of contact regions around surface asperities. With increasing load, the intensity of stress concentration at the largest interfacial flaw will first reach a critical level and the contact starts to fail by crack growth and coalescence. Under this circumstance, the adhesion strength is not optimal because only a small fraction of material is highly stressed at any instant of loading. From the robustness point of view,
it would be best to seek a design of material that allows the contact to fail not by crack propagation, but always by uniform detachment at the theoretical strength of adhesion, a concept termed as “flaw tolerance” (Gao et al., 2003, 2004, 2005; Gao and Chen, 2005). According to this concept, in an ideal flaw tolerant adhesion system, there should be no crack propagation and coalescence as the contact interface is pulled apart by uniform detachment.

For a single fiber on substrate (Fig. 2(a)), Gao and Yao (2004) has investigated the condition for flaw tolerant adhesion from the point of view of variations in contact shape. It was shown that there exist two extreme classes of contact shapes: one class (singular shapes) gives rise to a singular stress field at pull-off similar to that of an external crack (Fig. 2(b)) and the other class (optimal shapes) leads to a uniform stress at pull-off (Fig. 2(c)). For singular shapes, the pull-off force can be calculated according to the Griffith condition (Griffith, 1921) as

\[
P_{\text{crack}}^f = \pi R^2 \sqrt{\frac{8}{\pi} \left( \frac{E^* W_{\text{ad}}}{R} \right)^{1/2}},
\]

where \( W_{\text{ad}} \) denotes the work of adhesion and \( E^* = \left( 1 - v_f^2 \right)/E_f + \left( 1 - v_s^2 \right)/E_s \)^{-1}, \( E_f, E_s, v_f, v_s \) being Young’s moduli and Poisson’s ratios of the fiber and the substrate, respectively. For a gecko sticking to a solid surface, we assume \( E_s \gg E_f \), therefore \( E^* \approx E_f \left/ \left( 1 - v_f^2 \right) \right. \).

On the other hand, the pull-off force for optimal contact shapes (Fig. 2(c)) is

\[
P_{\text{th}}^f = \pi R^2 \sigma_{\text{th}},
\]

Fig. 2. Condition for flaw tolerant adhesion of a single fiber on a solid surface. (a) An elastic fiber is brought into contact with a substrate. Depending upon the shape of the fiber tip, the detachment process can occur either by (b) crack propagation (singular shapes) equivalent to an infinite crack external to the contact area or by (c) uniform detachment (optimal shapes) in which the stress at pull-off is uniformly distributed and equal to the theoretical adhesion strength \( \sigma_{\text{th}} \). The difference between the adhesive strength of these two failure modes vanishes as the size of the fibril is reduced to below a threshold \( R_{\text{cr}} = 8E^* W_{\text{ad}}/\pi \sigma_{\text{th}}^2 \), which is taken as the condition for flaw tolerant adhesion.
where $\sigma_{th}$ is the theoretical adhesion strength. Generally, $P_{f\text{crack}}$ is much smaller than $P_{f\text{th}}$. However, as the size of the fiber is reduced, the value of $P_{f\text{crack}}$ increases towards $P_{f\text{th}}$. At the critical size

$$R_{cr} = \frac{8}{\pi} \frac{E_f W_{ad}}{(1 - v_f^2)\sigma_{th}^2},$$

(3)

the pull-off force for the singular shapes predicted by the Griffith condition in Eq. (1) becomes identical to that for the optimal shapes in Eq. (2). Alternative derivations based on partial contact (Gao et al., 2005) or perfectly bonded contact (Glassmaker et al., 2005) lead to similar, but more relaxed, conditions on the fiber size. In this paper, we shall adopt Eq. (3) as the basic flaw tolerant condition for adhesion by a single fiber.

2.2. Energy dissipation in fibrillar structures

It can be seen from Eq. (3) that $R_{cr}$ is proportional to the work of adhesion $W_{ad}$ which is commonly taken as the differential surface energy $\Delta \gamma = \gamma_f + \gamma_s - \gamma_{fs}$ where $\gamma_f$, $\gamma_s$, $\gamma_{fs}$ denote the surface energies of fiber, substrate and fiber–substrate interface, respectively. However, this interpretation is correct only in the absence of other dissipation mechanisms. For slender elastic hairs in strong (flaw insensitive) adhesion with a solid surface, additional energy dissipation terms must be taken into account.

To illustrate this point, let us consider the adhesion between a larger fiber with a hairy tip surface in contact with a substrate, as shown in Fig. 3(a). Compared to the case shown in Fig. 2(a), the larger fiber in Fig. 3(a) contains a number of thinner fibrils on its tip surface, resulting in a two-leveled structure: an array of smaller fibrils on the tip surface of a larger fiber. For this structure, the work of adhesion for the larger fiber is no longer equal to $\Delta \gamma$ even though the small fibrils interact with the substrate only via van der Waals forces. To estimate the work of adhesion of the large fiber, we assume that the fibrils are thin enough to meet the condition for flaw tolerant adhesion. Fig. 3(b) plots the effective stress-separation relationship for the hairy surface, assuming Lennard–Jones (e.g., Greenwood, 1997) or Dugdale (1960) interaction law. While the stress-separation curves for two smooth surfaces are described by the van der Waals/Dugdale interaction laws at the atomic scale, the separation at the level of the larger fiber is strongly influenced by the elastic properties and geometry of the fibrils. For sufficiently long fibrils, the elastic deformation of the fibrils will make significant contributions to the separation process and adhesion failure occurs by an abrupt drop in stress near the theoretical strength of surface interaction. In this way, the strain energy stored in the fiber becomes part of the “cohesive energy” to be dissipated through dynamic snapping of the thin fibrils. In other words, the thin fibrils behave effectively as cohesive bonds for the larger fiber. The work of adhesion for the large fiber should therefore include the elastic energy stored in the fibrils when they are stretched to failure, i.e.

$$W_{ad} = (\Delta \gamma + \sigma_{th}^2 L/2E_f)\varphi.$$  

(4)

Here, $L$ is the length of the fibrils and $\varphi$ is the area fraction of the fibril array. The first term within the bracket represents the original van der Waals interaction energy and the second term is the elastic energy lost during dynamic snapping of the fibrils as they are detached from the substrate near the theoretical strength of van der Waals interaction. Eq. (4) also shows why it is important to optimize the adhesion strength of the lower level fibrils via...
size reduction: the strength of the lower scale fibrils directly contributes to the work of adhesion of the large fiber. Taking $\Delta g = 0.01 \, \text{J/m}^2$, $\sigma_{th} = 20 \, \text{MPa}$, $L = 100 \, \mu\text{m}$, $E_i = 1 \, \text{GPa}$, $\phi = 0.5$, the work of adhesion for the hairy tipped fiber is calculated to be $W_{ad} \approx 10 \, \text{J/m}^2$, a value much larger than $\Delta g$. Such enhancement in work of adhesion by fibrillar structures has been reported and/or discussed by Jagota and Bennison (2002), Persson (2003a), Gao et al. (2004) and Tang et al. (2005). Hence, slender hairs with large aspect ratio can significantly increase the work of adhesion and contribute to the
robustness of adhesion at higher structural levels. On the other hand, the length of the fibrils cannot be too long as there is an instability leading to fiber bunching as the aspect ratio of the fibrils increase. This is discussed in the following subsection.

2.3. Anti-bunching condition in fibrillar structures

In an array of slender hairs planted on a solid surface, the van der Waals interaction between neighboring fibers can cause them to bundle together (Sitti and Fearing, 2003; Hui et al., 2002; Geim et al., 2003; Persson, 2003a; Glassmaker et al., 2004; Gao et al., 2005). The anti-bunching condition is an important factor in the design of hairy adhesion structures. The exact form of the anti-bunching condition depends on the geometry of the fiber. For example, the anti-bunching condition for fibers of square cross-section has been derived by Hui et al. (2002) and Gao et al. (2005). In this paper, we focus on cylindrical fibers that have been considered by Glassmaker et al. (2004).

Consider two neighboring identical cylindrical fibers with circular cross-sections. When the separation $2w$ becomes small, the surface adhesive forces may cause them to bundle together, as shown in Fig. 4(a). The stability condition can be derived from the point of view of a maximum fiber length for spontaneous separation of two fibers sticking together (e.g., Gao et al., 2005). In other words, given fiber separation $w$ and radius $R$, there exists a critical length $L_{cr}$ beyond which lateral bunching of neighboring fibers becomes stable configurations. Glassmaker et al. (2004) have derived the critical length for bunching of cylindrical fibers as

$$L_{cr} = \left[ \frac{\pi^4 E_t R}{2^{11/2} \gamma_t (1 - \nu_t^2)} \right]^{1/12} \left[ \frac{12 E_t R^3 w^2}{\gamma_t} \right]^{1/4}.$$  \hfill (5)

Assuming that the fibers are distributed in a regular lattice pattern, one can relate the fiber separation $w$, radius $R$ to the area fraction $\varphi$ of a fiber array by

$$w = \left( \sqrt{\varphi_{\text{max}}/\varphi - 1} \right) R \quad (0 < \varphi < \varphi_{\text{max}}),$$  \hfill (6)

where $\varphi_{\text{max}}$ stands for the maximum area fraction of a given hair pattern. It can be shown that $\varphi_{\text{max}} = \pi/2\sqrt{3}$ for a triangular lattice (Fig. 4b), $\varphi_{\text{max}} = \pi/4$ for a square lattice

Fig. 4. Anti-bunching condition of a fibrillar structure. (a) Configuration of self-bunching in an array of fibers distributed in (b) triangular, (c) square or (d) hexagonal patterns.
(Fig. 4c) and $\varphi_{\text{max}} = \pi/3\sqrt{3}$ for a hexagonal lattice (Fig. 4d). Inserting Eq. (6) into Eq. (5) leads to
\[
L_{\text{cr}} = Ra \left( \frac{E_f R}{\gamma_f} \right)^{1/3} \left( \sqrt{\varphi_{\text{max}} / \varphi - 1} \right)^{1/2},
\]
where
\[
Ra = \left[ \frac{3^3 \pi^4}{2^5 (1 - \nu_f^2)} \right]^{1/12}.
\]

Eq. (7) has been derived for the lateral sticking between two neighboring fibrils. Similar analysis can also be carried out for other possible bunching configurations involving multiple neighboring fibers. We find that the critical fiber length for multiple fiber bunching is no less than that given by Eq. (7). It seems that the anti-bunching condition between two fibers is the most critical condition against bunching involving multiple fibers.

2.4. “Fractal gecko hairs”: bottom–up designed hierarchical fibrillar structures

Given that the work of adhesion can be increased to a larger value by adopting a “hairy” structure (Jagota and Bennison, 2002; Persson, 2003a; Gao et al., 2004; Tang et al., 2005), the critical length for flaw tolerant adhesion can also be extended to a larger scale, according to Eq. (3). Meanwhile, the increase in work of adhesion with each level of added hierarchy should be limited by the maximum length of the fibers allowed by the anti-bunching condition. In other words, bunching between fibers provides an upper limit on how much the flaw tolerant length scale can be extended by one level of hierarchy. In order to achieve flaw tolerant adhesion at macroscopic length scales, multiple levels of hierarchy may be needed. To demonstrate the principle of flaw tolerance via structure hierarchy, we propose a “fractal gecko hairs” model, in which a hierarchical fibrillar structure is made from multiple levels of self-affine “brush” structures, as shown in Fig. 5. In this fractal structure, the tips of fibers at each level of hierarchy are assumed to be coated with a “brush” structure consisting of smaller fibrils from one level below. The flaw tolerance and anti-bunching conditions are applied to all hierarchical levels from bottom and up to ensure robustness and stability at all levels. That is, the robustness principle of flaw tolerance and the stability principle of anti-bunching are used to determine the fiber geometry at different scales. The bottom–up construction of the desired hierarchical structure is described in some detail below.

At the lowest level of hierarchy, the failure process is governed by the van der Waals interaction between the smallest fibers (ultrastructure) and a solid surface. In this case, the maximum fiber radius ensuring flaw tolerant adhesion is given by
\[
R_1 = \frac{8 \Delta \gamma E_f}{\pi (1 - \nu_f^2) \sigma_{\text{th}}^2},
\]
where the work of adhesion is simply equal to the surface energy $\Delta \gamma$ due to van der Waals interaction and $\sigma_{\text{th}}$ is the theoretical strength of van der Waals forces.
In light of the anti-bunching condition of Eq. (7), the maximum fiber length of the bottom level can be expressed as a function of the area fraction $\varphi_1$ of this level as

$$L_1(\varphi_1) = R_1 \pi \left( \frac{E_f R_1}{\gamma_f} \right)^{1/3} \left( \sqrt{\frac{\varphi_{\text{max}}}{\varphi_1}} - 1 \right)^{1/2}. \quad (9)$$

With these parameters, the work of adhesion associated with the next (second) level is given by

$$W_{\text{ad}}^2(\varphi_1) = \left( \frac{\sigma_{\text{th}}^2 L_1}{2 E_f} + \Delta_y \right) \varphi_1, \quad (10)$$

which is a function of the area fraction $\varphi_1$. This function exhibits a maximum at a specific value of $\varphi_1$ due to the opposing trends of variation of the parameters $L_1$ and $\varphi_1$: denser fibers with larger $\varphi_1$ require smaller $L_1$ for stability against bunching. Therefore, we can choose the fiber area fraction $\varphi_1$ to maximize the work of adhesion at the next level according to Eq. (10). After $\varphi_1$ is calculated, the fiber length $L_1$ is immediately determined by Eq. (9). In this way, all the structural parameters characterizing the first level $R_1, L_1, \varphi_1$ have been determined. In addition, the work of adhesion for the second level $W_{\text{ad}}^2$ is given by Eq. (10).

We now advance to the design of the second (next) level. The fiber radius is again chosen to ensure flaw tolerant adhesion,

$$R_2 = \frac{8 W_{\text{ad}}^2 E_f}{\pi (1 - v_f^2)(S_2)^2} = \frac{8 W_{\text{ad}}^2 E_f}{\pi (1 - v_f^2)(\varphi_1 \sigma_{\text{th}})^2}, \quad (11)$$

Fig. 5. Bottom–up design scheme of a hierarchical fibrillar structure. At each level, the fibers depend on smaller fibrils from the lower hierarchical levels as effective “adhesive bonds” with a surface. Interestingly, the fibers themselves act as “adhesive bonds” for larger fibers from higher hierarchical levels.
where $S_2 = \varphi_1 \sigma_{th}$ is the effective adhesion strength of the second level. Similarly, the anti-bunching condition allows the fiber length to be determined as a function of the area fraction $\varphi_2$ as

$$L_2(\varphi_2) = R_2 \sqrt{\frac{E_f R_2}{\gamma_f}} \left( \sqrt{\frac{\varphi_{\text{max}}}{\varphi_2} - 1} \right)^{1/2},$$

(12)

upon which the work of adhesion for the third level can be determined,

$$W_{3}^{\text{ad}}(\varphi_2) = \left( W_2^{\text{ad}} + \frac{(S_2)^2 L_2^2}{2E_f} \right) \varphi_2 = \left( W_2^{\text{ad}} + \frac{(\varphi_1 \sigma_{th})^2 L_2^2}{2E_f} \right) \varphi_2.$$  

(13)

Next, the area fraction $\varphi_2$ is determined by maximizing $W_{3}^{\text{ad}}(\varphi_2)$. Once $\varphi_2$ is known, the fiber length $L_2$ is determined from Eq. (12). Hence all the structural parameters, $R_2$, $L_2$, $\varphi_2$, for the second hierarchical level, as well as the work of adhesion $W_{3}^{\text{ad}}$ for the third level, have been determined.

A general iterative procedure can now be formulated to determine the structural parameters at all hierarchical levels, starting from the lowest level. Assuming we have completed the design from the first to $(n-1)$th levels so that $R_i$, $L_i$, $\varphi_i$, $W_i^{\text{ad}}$ ($i = 1, 2, \ldots, n-1$) as well as $W_n^{\text{ad}}$ have been determined, for the $n$th level ($n > 1$), the (maximum) fiber radius ensuring flaw tolerant adhesion is given by

$$R_n = \frac{8 W_n^{\text{ad}} E_f}{(1 - v_f^2)(S_n)^2} = \frac{8 W_n^{\text{ad}} E_f}{(1 - v_f^2)\pi(\varphi_{th} \Phi_{n-1})^2},$$

(14)

where

$$S_n = \sigma_{th} \Phi_{n-1}, \quad \Phi_{n-1} = \varphi_1 \varphi_2 \ldots \varphi_{n-1} = \prod_{i=1}^{n-1} \varphi_i,$$

(15)

is the effective adhesion strength of the $n$th level. The (maximum allowable) fiber length of the $n$th level can then be expressed, according to the anti-bunching condition, as a function of the area fraction $\varphi_n$,

$$L_n(\varphi_n) = \alpha R_n \left( \sqrt{\frac{\varphi_{\text{max}}}{\varphi_n} - 1} \right)^{1/2} \left( \frac{E_f R_n}{\gamma_f} \right)^{1/3}.$$

(16)

The work of adhesion for the $(n+1)$th level is

$$W_{n+1}^{\text{ad}}(\varphi_n) = \left( W_n^{\text{ad}} + \frac{(S_n)^2 L_n}{2E_f} \right) \varphi_n = \left( W_n^{\text{ad}} + \frac{(\varphi_{th} \Phi_{n-1})^2 L_n}{2E_f} \right) \varphi_n.$$  

(17)

The area fraction for the $n$th level $\varphi_n$ can now be determined by maximizing $W_{n+1}^{\text{ad}}(\varphi_n)$, upon which $L_n$ and $W_{n+1}^{\text{ad}}$ can be readily calculated. This iterative, bottom-up design procedure can be repeated until the desired size scale for flaw tolerant adhesion is reached. Upon the knowledge of the fiber radius and area fraction of each level, we can calculate the number of fibrils on the tip of a fiber at the next higher level,

$$N_n^f = \varphi_n (R_{n+1}/R_n)^2,$$

(18)

as well as the net pull-off force at each hierarchical level,

$$F_n = \pi R_n^2 S_n.$$  

(19)
Fig. 6. Variations of (a) fiber radius $R_n$, (b) fiber length $L_n$, (c) area fraction $\varphi_n$, (d) work of adhesion $W^\text{ad}_n$, (e) adhesion strength $S_n$, (f) pull-off force $F_n$ and (g) the number of fibers $N_n$ as a function of the hierarchical level $n$. 
Fig. 6 shows the calculated hierarchical fibrillar structures following the bottom–up design procedure described above. In the calculations, we have taken the material properties of keratin as $E_f = 1.0$ GPa, $\nu_f = 0.3$, $\Delta \gamma = 10$ mJ/m$^2$, $\gamma_f = 5$ mJ/m$^2$ and $\sigma_{th} = 20$ MPa. Three lattice patterns, triangular, square and hexagonal, for the fiber array are considered. As shown in Figs. 6(a) and (b), both the fiber radius and length increase exponentially with the hierarchy level. Under the selected parameters, the critical fiber radius of flaw tolerant adhesion is only around 100 nm at the lowest level of structure. With hierarchical design, the flaw tolerant radius increases to 1 µm with 2 levels, 1 mm with 3 levels, 1 m with 4 levels of hierarchy. With 8 levels, the dimension of flaw tolerant radius has reached $10^{26}$ m, which is an astronomical size! These calculations demonstrate the enormous potential of bottom–up hierarchy for flaw tolerant adhesion. Fig. 6(c) displays the variation of the area fraction with the number of hierarchy levels. Interestingly, the area fraction converges to a constant after the third hierarchy level for each fiber layout pattern. Fig. 6(d) shows the work of adhesion at different hierarchical levels. In the first 6 levels, the triangular fiber pattern exhibits higher work of adhesion than the other two patterns. With further increase in hierarchy levels, this advantage is taken over by the hexagonal fiber pattern. Fig. 6(e) shows the effective adhesion strength which decreases and asymptotically approaches zero with the increasing hierarchy level. On the other hand, the net pull-off force, as shown in Fig. 6(f), is seen to increase exponentially with the hierarchy level. Fig. 6(g) illustrates the number $N_n^f$ of fibrils on the tip of a fiber at the next level. We see that $N_n^f$ increases sharply with increasing hierarchy levels. Most results in Fig. 6 are presented in the normalized form. The quantitative estimates based on the assumed materials properties are tabulated in Table 1.

It should be of interest to make a comparison between our calculated results with the observed hierarchical structure of gecko. Under the selected parameters, our results show that the diameter and length of the first level fiber are 140 nm and 1.37 µm. These values are not inconsistent with the dimension of the topmost spatula hairs (stalk) of Tokay gecko (Gekko gecko) which is around 100–200 nm wide and 0.5–3 µm long (Williams and Peterson, 1982; Autumn et al., 2000). The dimension of the second level of our bottom–up constructed structure is around 7.56–11.34 µm wide and 286–491 µm long, depending upon the pattern of the fiber array, while the size of a seta on gecko’s feet is about 5 µm in width and 110 µm in length (Williams and Peterson, 1982).

In addition, our calculation predicts that the number of the lowest level fibrils accommodated by a fiber of the second level is around 1539–2309, which is qualitatively similar to the observation of 100–1000 spatulae/seta (Autumn and Peattie, 2002). Furthermore, from our calculated results we evaluate the density $N_n^f/\pi R_3^2$ of the second level fiber to be $11,494$, $7363$, $3439$ mm$^{-2}$ for triangular, square and hexagonal patterns, respectively. This is also comparable to the observed density of seta around 14,400 mm$^{-2}$ (Autumn and Peattie, 2002). Therefore, it seems that gecko only adopts a few levels of hierarchical fibrillar structures to achieve robust adhesion. A question then is why nature has not evolved more hierarchical levels, thus larger adhesion species heavier than gecko? A possible answer to this question is addressed in the next subsection.

\footnote{These values are estimated from the micrographs in the references.}
Table 1
Calculated geometrical and mechanical properties of bottom-up designed fractal gecko hair structure

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<th>Re (m)</th>
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2.5. Fiber fracture: an upper limit on flaw tolerant adhesion design via multiscale fibrillar structures

In the preceding discussions, we have focused on failure along an adhesion interface and implicitly assumed that the fibers themselves do not fracture. In practice, as the adhesion strength is enhanced by introducing hierarchical fibrillar structures, the fracture of fibers eventually rises to become the dominant mode of failure at the system level. In other words, a robust adhesion system must be robust against not only adhesion failure but also fiber fracture.

Consider a single fiber at hierarchy level \( n \). A penny-shaped crack is introduced in the center of the cross-section as a possible internal flaw. Other configurations of crack-like flaws, such as edge/corner cracks/singularities, can be considered without affecting the basic idea. The maximum tensile stress that this cracked fiber can sustain can be determined from Griffith’s criterion for crack growth as,

\[
\sigma_{\text{max}}^n = \sqrt{\frac{E_I\Gamma_I}{R_n} \frac{\sqrt{\pi R_n/2a}}{g(a/R_n)}},
\]
where $a$ is the crack radius, $\Gamma_f$ is the fracture energy and

$$g \left( \frac{a}{R_n} \right) = \frac{1 - 0.5a/R_n + 0.148(a/R_n)^3}{\sqrt{1 - a/R_n}}$$

(21)

is a geometrical parameter (Tada et al., 2000). Considering a crack half the size of the fiber, i.e. $a/R_n = 0.5$, Eq. (20) can be further reduced to

$$\sigma_n^{\text{max}} = 1.63 \sqrt{E_r^* \Gamma_f / R_n}.$$  (22)

The relative significance of fiber fracture can be measured by a comparison between $\sigma_n^{\text{max}}$ and the effective adhesion strength $S_n$ at the $n$th hierarchical level. If $\sigma_n^{\text{max}} > S_n$, adhesion failure is regarded as the principal failure mode and further increase in hierarchical levels can be considered. On the other hand, if $\sigma_n^{\text{max}} < S_n$, fiber fracture is regarded as the dominant failure mode, which imposes an upper limit on the hierarchical design. Taking $\Gamma_f = 5 \text{ J/m}^2$ and $E_r^* = 1 \text{ GPa}$, we compare $\sigma_n^{\text{max}}$ and $S_n$ for the fractal hair structures constructed above. As shown in Fig. 7, for triangular and square fiber layout, only fibers within the first two levels satisfy the condition $\sigma_n^{\text{max}} > S_n$; for the hexagonal layout, this condition is satisfied for the first three levels. Hence, although there is no upper bound for flaw tolerant adhesion via fractal hairs design, crack-like flaws in the hairs themselves would impose a practical limit for the usefulness of this strategy.

3. Releasable adhesion

For geckos and insects, robust adhesion alone is insufficient for survival as these animals also need to move swiftly on walls and ceilings; the reversibility of attachment is just as important as the attachment. A conceivable strategy for reversible adhesion is to design an orientation-controlled switch between attachment and detachment, with adhesion strength...
varying strongly with the direction of pulling. An ideal scenario of robust and releasable adhesion is that the adhesion strength would be maintained near the theoretical strength when pulled in some range of directions, but then dramatically reduced when pulled in another range of directions. The switch between attachment and detachment can thus be accomplished simply by changing the pulling angles (e.g., by exerting different muscles). Some known examples of anisotropic adhesion systems in which the pull-off force varies strongly with the direction of pulling include an elastic tape on substrate (Kendall, 1975; Spolenak et al., 2005; Huber et al., 2005) and a single seta of gecko sticking on a wall (Autumn et al., 2000; Gao et al., 2005). Here we show that such behavior can actually be generalized to three-dimensional elastic solids as long as there is sufficiently strong elastic anisotropy.

3.1. Orientation-dependent adhesion strength of an elastic tape

For an elastic tape adhering on a substrate, as shown in Fig. 7(a), Kendall (1975) calculated the critical force required to peel the tape off the substrate, which can be written as

\[
F = E_{tp}hd \left[ \sqrt{(1 - \cos \phi)^2 + 2\Delta \gamma/E_{tp}h} - (1 - \cos \phi) \right]
\]

\[
= \frac{2\Delta \gamma d}{\sqrt{(1 - \cos \phi)^2 + 2\Delta \gamma/E_{tp}h} + (1 - \cos \phi)}.
\]

Here, \(E_{tp}\) denotes Young’s modulus of the tape while \(h, d\) stand for the thickness and width of the tape, respectively. For a given elastic tape, Eq. (23) indicates that the peel-off force varies with the pulling angle \(\phi\). Taking \(\Delta \gamma/E_{tp}h = 10^{-4}\) (\(\Delta \gamma = 0.01 \text{ J/m}^2, E_{tp} = 1.0 \text{ GPa}, h = 100 \text{ nm}\)), Fig. 8(a) plots the variation of the normalized peel-off force and its projection normal to the contact interface as a function of the pulling angle \(\phi\). It is evident that the peel-off force varies strongly with the pulling angle. Under the selected parameter values, the normal projection of the peeling force exhibits a maximum value around \(\phi \approx 75^\circ\).

![Fig. 8. Releasable adhesion in one-dimensional structures. Variation of the pull-off force as a function of the pulling angle for (a) an elastic tape and (b) a single seta of gecko on a surface (adapted from Gao et al., 2005).](image-url)
3.2. Orientation-dependent adhesion strength of a single seta

Autumn et al. (2000) reported experimental data that the detachment force of a single seta of gecko on a surface strongly depends on the orientation of pulling, with the peak value achieved as the seta is pulled at an inclined angle of $\phi = 30^\circ$ with respect to the tangent of the surface. Motivated by this experiment, Gao et al. (2005) performed finite element calculations of the pull-off force of a single seta and, as shown in their results plotted in Fig. 8(b), demonstrated theoretically that the pull-off force of gecko’s seta strongly varies with the pulling orientation with the maximum value achieved around $\phi = 30^\circ$.

In the case of single contact by an elastic tape or seta, the anisotropic behavior of the pull-off force can be attributed to the asymmetric alignment and slender structure of the contacting object. While this behavior suggests that the pull-off force of a single hair in contact can be controlled by pulling in different directions, an open question is whether the adhesive strength of a large array of fibers or a macroscopic attachment pad in contact with a rough surface would show similar behaviors. To address this question, we shall consider the issue of releasable adhesion from the point of view of continuum interfacial failure mechanics. We use theoretical modeling and numerical simulations to show that strong elastic anisotropy on the continuum level, achieved via fibrillar microstructures or some other alternative microstructural design), plays a key role in releasable adhesion: a strongly anisotropic elastic solid also exhibits a strong orientational dependence of the pull-off force, similar to the behavior of a single seta studied by Gao et al. (2005).

3.3. Orientation-dependent adhesion strength of an anisotropic elastic material

To illustrate the intrinsic orientation dependence of adhesion strength of an anisotropic elastic material in contact with a rough surface, we consider the linear elastic plane-strain problem shown in Fig. 9(a), where a transversely isotropic elastic half-space ($y \geq 0$) is brought to contact with a rigid substrate. A plane-strain interfacial crack of size $2a$ is used.
to represent random contact flaws due to surface roughness or contaminants. Although the actual adhesion strength depends on the crack size, the ratio between the maximum and minimum pull-off stresses as the pulling angle varies will be shown to be independent of the crack geometry and can be used as a measure of the releasability of adhesion.

In this interfacial crack model, the longitudinal direction of the material \((v_0 \text{ axis})\) is tilted at an angle \(\theta\) from the tangent of the substrate surface \((x\text{-axis})\). A remote uniaxial tensile stress \(\sigma^\infty\) is applied at an angle \(\phi\) with respect to the \(x\text{-axis}\). The transversely isotropic material is characterized by five independent elastic constants: \(E_t, E_l, v_t, v_l\) and \(\mu\). \(E_t\) and \(E_l\) stand for the transverse \((x_0 \text{ direction})\) and longitudinal \((y_0 \text{ direction})\) Young’s moduli; \(v_t, v_l\), are Poisson’s ratios associated with transverse \((x_0 \text{ direction})\) and longitudinal \((y_0 \text{ direction})\) loading; \(\mu\) denotes the shear modulus in the \(x_0\)–\(y_0\) plane.

We are interested in the pull-off stress of the above adhesion system as a function of the pulling direction. This problem can be solved as a classical interfacial crack between two dissimilar anisotropic elastic solids (Gotoh, 1967; Willis, 1971; Ting, 1986; Suo, 1990; Gao et al., 1992; Hwu, 1993). The energy release rate induced by remotely applied normal and shear stresses \((\sigma_{xy}^\infty, \sigma_{yy}^\infty)\) is calculated to be (see Appendix A)

\[
G = \frac{\pi a(1 + 4\varepsilon^2)}{4 \cosh^2 \pi \varepsilon} \left[ D_{22}(\sigma_{xy}^\infty \cos \theta + \sigma_{yy}^\infty \sin \theta)^2 + D_{11}(\sigma_{xy}^\infty \sin \theta - \sigma_{yy}^\infty \cos \theta)^2 \right],
\]

where

\[
\varepsilon = \frac{1}{2\pi} \ln \left| \frac{1 + \beta}{1 - \beta} \right|, \quad \beta = |W_{21}(D_{11}D_{22})^{-1/2}|,
\]

\[
D_{11} = \frac{1}{E_l} \left[ \frac{E_t}{E_l}(1 - v_t^2)^{1/2} \left( \frac{E_l}{\mu} + 2 \sqrt{\left(1 - v_t^2\right) - v_l(1 + v_l)} \right) \right]^{1/2},
\]

\[
D_{22} = \frac{1}{E_l} \left[ \left(1 - v_t^2\right)^{1/2} \left( \frac{E_l}{\mu} + 2 \sqrt{\left(1 - v_t^2\right) - v_l(1 + v_l)} \right) \right]^{1/2},
\]

\[
W_{21} = -\frac{1}{E_l} \sqrt{\left(1 - v_l^2\right) \left(1 - v_t^2\right)} - (1 + v_l)v_l \sqrt{E_l}. \tag{25}
\]

For the uniaxial pulling stress \(\sigma^\infty\) applied at an inclined angle \(\phi\), as shown in Fig. 9(a), the components \(\sigma_{xy}^\infty\) and \(\sigma_{yy}^\infty\) can be expressed as

\[
\sigma_{xy}^\infty = \sigma^\infty \sin \phi \cos \phi, \quad \sigma_{yy}^\infty = \sigma^\infty \sin \phi \sin \phi. \tag{26}
\]

Substituting (26) into (24) and then applying Griffith’s criterion for crack initiation \(\gamma = W_{ad}\) lead to the adhesion strength

\[
\sigma^\infty_{cr}(\theta, \phi) = \frac{\sqrt{W_{ad}/\pi a}}{\sin \phi \sqrt{C[D_{22} \cos^2(\theta - \phi) + D_{11} \sin^2(\theta - \phi)]}}, \tag{27}
\]

where

\[
C = \frac{(1 + 4\varepsilon^2)}{4 \cosh^2 \pi \varepsilon}. \tag{28}
\]
Given material constants and the anisotropy direction \( \theta \), Eq. (27) indicates that the adhesion strength varies as a function of the pulling angle \( \phi \). To calculate the critical (maximum and minimum) values as well as the corresponding directions, we solve equation \( \partial \sigma_{cr}^2(\theta, \phi)/\partial \phi = 0 \) and obtain

\[
\frac{1 + D_{22}/D_{11}}{1 - D_{22}/D_{11}} \cos \phi = \cos(3\phi - \theta), \quad D_{22}/D_{11} = \sqrt{\frac{E_i(E_i - v_i^2E_l)}{E_l(1 - v_i^2)}}. \tag{29}
\]

If Young's modulus in the longitudinal direction (e.g., along a fiber array) is much larger than that in the transverse direction (e.g., transverse to the fiber direction), i.e. \( E_l/E_i \gg 1 \), Eq. (29) has two roots

\[
\phi_1 = \theta, \quad \phi_2 = \theta/2 + \pi/2, \tag{30}
\]
corresponding to the directions of the maximum and minimum pull-off stresses, respectively. The adhesion releasability thus can be measured by the ratio of the maximum to the minimum pull-off stresses:

\[
\frac{(\sigma_{cr}^\infty)_{\text{max}}}{(\sigma_{cr}^\infty)_{\text{min}}} = \frac{(1 + \cos \theta)}{2 \sin \theta} \left( \frac{D_{11}}{D_{22}} \right)^{1/2} = \frac{(1 + \cos \theta)}{2 \sin \theta} \left[ \frac{E_l^2(1 - v_i^2)}{E_i(E_i - v_i^2E_l)} \right]^{1/4}. \tag{31}
\]

For small Poisson’s ratios, Eq. (31) suggests that the releasability of adhesion mainly depends on the stiffness ratio \( E_l/E_i \) and the anisotropy direction \( \theta \). The stronger the anisotropy, the higher the releasability of adhesion. Assuming \( v_i = v_t = 0.3, \theta = 30^\circ \) and \( E_l/E_i = 10^4 \), Fig. 9(b) plots the normalized pull-off stress as a function of the pulling angle \( \phi \). We can see that the elastic anisotropy causes about an order of magnitude change in adhesion strength as the pulling angle varies. A switch between attachment and detachment can thus be accomplished just by shifting the pulling angle between these two directions. In contrast, the adhesion strength for an isotropic material with \( E_l = E_i \) and \( v_i = v_t \) is much less sensitive to the pulling direction. We conclude that strong elastic anisotropy can result in an orientation-controlled switch between attachment and detachment.

One might note that Eq. (27) implies an infinite adhesion strength in the limit of \( \phi = 0 \). This is caused by the assumed loading by a uniaxial tensile stress. Actually, the limit \( \phi = 0 \) should be characterized as sliding under an applied shear stress. If, instead of pulling, we apply a remote shear stress \( \sigma_{xy}^\infty \), the critical shear stress becomes

\[
(\sigma_{xy}^\infty)_{\text{cr}} = \frac{\sqrt{W_{ad}/\pi a}}{\sqrt{C(D_{22}\cos^2 \theta + D_{11} \sin^2 \theta)}}, \tag{32}
\]

which can be reduced to

\[
(\sigma_{xy}^\infty)_{\text{cr}} = \frac{2\sqrt{W_{ad}/\pi a}}{\sqrt{CD_{11}}} \tag{33}
\]

when \( D_{22} \ll D_{11} \) and \( \theta = 30^\circ \), and to

\[
(\sigma_{xy}^\infty)_{\text{cr}} = \frac{\sqrt{W_{ad}/\pi a}}{\sqrt{CD_{11}}} \tag{34}
\]

when \( D_{22} = D_{11} \) for the isotropic case.
3.4. Orientation-dependent adhesion strength of an attachment pad: numerical simulation

To further verify the principle of orientation-controlled adhesion switch via strong elastic anisotropy, we have also performed numerical simulations of the adhesion of a strongly anisotropic attachment pad (mimicking the hairy structured tissue on gecko’s feet) via a general-purpose finite element code Tahoe\textsuperscript{2} with specialized cohesive surface elements for modeling adhesive interactions between two surfaces. The constitutive relation for the cohesive surface elements is specified in terms of a relation between the traction and separation across the contact interface. Tahoe supports a number of traction-separation laws including the Tvergaard–Hutchinson law (Tvergaard and Hutchinson, 1992) and the Xu–Needleman law (Xu and Needleman, 1994). In the present simulations, we adopt the Tvergaard–Hutchinson law based on the following interaction potential:

\[
\Phi(\lambda) = \delta_{cn} \int_{0}^{\lambda} \tilde{\phi}(\tilde{\lambda}) \, d\tilde{\lambda},
\]

where

\[
\tilde{\lambda} = \sqrt{\left(\frac{A_n}{\delta_{cn}}\right)^2 + \left(\frac{A_t}{\delta_{ct}}\right)^2}.
\]

Here, \(A_n\) denotes the normal separation and \(A_t\) the tangential separation; \(\delta_{cn}\), \(\delta_{ct}\) are the corresponding critical separations. The force function \(\tilde{\phi}\) is taken to be tri-linear,

\[
\tilde{\phi}(\lambda) = \begin{cases} 
\frac{\sigma_{\text{max}} \lambda}{A_1} & (\lambda < A_1), \\
\sigma_{\text{max}} & (A_1 < \lambda < A_2), \\
\sigma_{\text{max}}(1 - \lambda)/(1 - A_2) & (A_2 < \lambda < 1),
\end{cases}
\]

where \(A_1\) and \(A_2\) define the values of \(\lambda\) at which the cohesive force reaches the peak. Taking the partial derivatives of the potential with respect to the normal and tangential separations gives the normal and tangential tractions as

\[
T_n = \frac{\partial \Phi}{\partial A_n} = \frac{A_n}{\delta_{cn}} \frac{\tilde{\phi}(\tilde{\lambda})}{\tilde{\lambda}},
\]

\[
T_t = \frac{\partial \Phi}{\partial A_t} = \frac{\delta_{cn} A_t}{\delta_{ct} \delta_{ct}} \tilde{\phi}(\tilde{\lambda}).
\]

Clearly, the Tvergaard–Hutchinson law takes into account of both normal and tangential tractions with a constant work of adhesion

\[
W_{\text{ad}} = 1/2(1 + A_2 - A_1)\sigma_{\text{max}}\delta_{cn}.
\]

The simulation system consists of a plane-strain anisotropic (transversely isotropic) elastic pad adhering to a rigid substrate with a crack situated at the central region of the contact interface (representing an adhesion flaw due to surface roughness), as shown in Fig. 10(a). A displacement-controlled load is applied on the upper surface. At a given displacement, summation of all the nodal forces on the upper surface gives the pulling force \(F\) with components \(F_x, F_y\). The pulling angle is then calculated via \(\phi = \tan^{-1}(F_y/F_x)\).
Periodic boundary conditions are applied on the left and right sides of the simulation domain. For comparison, both isotropic case and anisotropic case are considered. The material constants and potential parameters for each simulation case are listed in Table 2 where we adopt the calculated $S_3$ and $W_{ad}$ of the triangular hair pattern (see Table 1) as the effective adhesion strength and work of adhesion in simulating the detachment process of the pad.

Fig. 10(b) plots the normalized pull-off stress $F_c(\phi)/(A\sigma_{max})$ as a function of the pulling angle $\phi$. In the anisotropic case, saturation of adhesion strength is observed in the vicinity of $\phi = \theta = 30^\circ$, corresponding to a plateau of the curve in the range of $20^\circ < \phi < 40^\circ$. If the pulling angle deviates from this range in either direction, the adhesion strength decreases quickly to a lower plateau, illustrating the anisotropy-induced releasability of adhesion. This two-plateau adhesion strength is ideal for rapid switch between attachment and detachment during animal movement. The ratio between the maximum and minimum strengths reaches a factor of four for the given geometry, giving rise to significant releasability. In contrast, for the isotropic cases, no variation in pull-off force is observed as the pulling angle varies. Therefore, we conclude that strong elastic anisotropy leads to releasable adhesion via an orientation-controlled switch between strong and weak adhesion.

### Table 2

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<tr>
<td><strong>Isotropic case</strong></td>
</tr>
<tr>
<td>$E = 1.0$ GPa, $\nu = 0.3$</td>
</tr>
<tr>
<td><strong>Anisotropic case</strong></td>
</tr>
<tr>
<td>$E_i = 0.1$ MPa, $E_l = 1.0$ GPa, $\nu_i = \nu_l = 0.3$, $\mu = 10$ MPa, $\theta = 30^\circ$</td>
</tr>
<tr>
<td><strong>Parameters for Tvergaard–Hutchinson model</strong></td>
</tr>
<tr>
<td>$\Lambda_1 = 0.1$, $\Lambda_2 = 0.9$, $\sigma_{max} = 5.44$ MPa, $\delta_{cn} = \delta_{ct} = 1.687$ µm. ($W_{ad} = 8.26$ J/m²)</td>
</tr>
</tbody>
</table>

4. Summary and discussion

In this paper, we have studied the basic principles of robust and releasable adhesion in the hierarchical structures of gecko. The work has been inspired by comparative studies of biological attachment systems in nature. For robust adhesion, we use a bottom–up...
designed fractal hair structure as a model to demonstrate that hierarchical fibrillar structures can lead to robust adhesion at macroscopic scales. Barring fiber fracture, we show that the fractal gecko hairs system can tolerate crack-like flaws without size limit. However, in practice, as the adhesion strength is enhanced by structural hierarchy, fiber fracture ultimately becomes the dominant failure mechanism and places an upper bound on the size scale of flaw tolerant adhesion design via fibrillar structures. In the optimal case, only an appropriate number of hierarchical levels should be introduced so that the adhesion interface achieves identical or similar strength level as the hairs. For releasable adhesion, we have shown that strong elastic anisotropy allows the adhesion strength to vary strongly with the direction of pulling. This orientation-dependent pull-off force enables robust attachment in the stiff direction of the material to be released by pulling in the soft direction. This strategy, conveniently summarized as a “stiff-adhere, soft-release” principle, can be understood in a simple way as follows. When pulled in the stiff direction, less elastic energy can be stored in the material (much like a stiff spring can store less energy compared to a soft spring), leading to lower energy release rate to drive random crack-like flaws induced by surface roughness. On the other hand, much more elastic energy can be stored in the material when pulled in the soft direction, especially when the material is strongly anisotropic, leading to much higher energy release rate to drive the roughness induced crack-like flaws.

The complex hierarchical structures in biology provide a rich source of inspirations for physical sciences and industrial applications. The concepts developed in this paper should be of general value in understanding biological attachment devices and the design of synthetic adhesive systems in engineering (e.g., Geim et al., 2003; Northen and Turner, 2005). While we usually do not expect to capture all the bio-complexities in simple models, it is often worthwhile to break a complex problem into smaller, more comprehensible sub-problems that can be further dissected and understood using simple mechanics principles. Here, we have considered the effects of hierarchical energy dissipation and elastic anisotropy on robust and releasable adhesion. Many other important aspects of the problem, such as viscoelasticity and large nonlinear deformation have not been taken into account. Much further work will be needed to advance our current understanding of bio-adhesion mechanisms. The studies on such problems should be of interest not only to the mechanics community but also to a variety of other disciplines including materials science, biology and nanotechnology.

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Appendix A. Derivation of the energy release rate

A general solution for the interfacial crack problem between two dissimilar anisotropic materials has been investigated by many authors (see, e.g., Ting, 1996; Suo, 1990; Gao
et al., 1992; Hwu, 1993) based on the Stroh formalism (Stroh, 1958). In this field of research, several different groups of notations have been used in the literature. Here, we summarize the results immediately relevant to our present study using the notations in Hwu (1993).

Consider a finite interfacial crack between two dissimilar anisotropic elastic solids. The stress intensity factor at the right crack tip induced by remote loading \( t^\infty = [\sigma_{xy}^\infty \sigma_{yy}^\infty \sigma_{yz}^\infty]^T \) can be expressed as

\[
K = \left[ \begin{array}{c}
K_{II} \\
K_I \\
K_{III}
\end{array} \right] = \sqrt{\pi a A} \langle (1 + 2i\varepsilon_2)(2a/l)^{-i\varepsilon_2} \rangle A^{-1} t^\infty, \quad (A.1)
\]

where the angular bracket \( \langle \{ \} \rangle \) stands for the diagonal matrix, i.e. \( \langle \{ f_2 \} \rangle = \text{diag} [f_1, f_2, f_3] \); \( \text{Im}(\cdot) \) denotes the imaginary part of a complex variable; \( 2a \) is the width of the interface crack; \( l \) is a length parameter which can be chosen arbitrarily; \( \delta_2 \) and \( \lambda_2 \) (\( \alpha = 1, 2, 3 \)) are eigenvalues and eigenvectors of the following problem:

\[
(M^* + e^{2i\delta_2} \bar{M}^*)^\alpha_\lambda = 0. \quad (A.3)
\]

Here, \( \bar{M}^* \) is the complex conjugate of the bimaterial matrix \( M^* \) which is defined as

\[
M^* = D - iW, \quad (A.4)
\]

\[
D = L_1^{-1} + L_2^{-1}, \quad W = S_1 L_1^{-1} - S_2 L_2^{-1}, \quad (A.5)
\]

where \( S, L \) are the Barnett–Lothe tensors determined by the material constants (Ting, 1996); subscripts “1” and “2” are used to denote quantities pertaining to the materials 1 and 2. The explicit solution to the eigenvalues of problem Eq. (A.3) have been obtained by Ting (1986) as

\[
\delta_2 = -\frac{1}{2} + i\varepsilon_2 \quad (\alpha = 1, 2, 3),
\]

\[
\varepsilon_1 = \varepsilon = \frac{1}{2\pi} \ln \frac{1 + \beta}{1 - \beta}, \quad \varepsilon_2 = -\varepsilon, \quad \varepsilon_3 = 0, \quad \beta = \left[ -\frac{1}{2} \text{tr}(WD^{-1})^2 \right]^{1/2}, \quad (A.6)
\]

where “tr” stands for the trace of a matrix.

The influence of the material properties on the solutions is reflected through the Barnett–Lothe tensors \( S, L \) whose matrix expressions are composed of elastic constants (Dongye and Ting, 1989). For a given bimaterial, once the matrices \( S, L \) of each material are obtained, the stress intensity factor can be deduced from Eqs. (A.1) to (A.6) and the corresponding energy release rate can be evaluated according to the formula

\[
G = \frac{1}{2} K^T E K, \quad (A.7)
\]

where

\[
E = D + WD^{-1}W. \quad (A.8)
\]
Substitution of Eq. (A.1) into Eq. (A.7) yields

\[
G(t^\infty) = \frac{\pi a}{4} [(t^\infty)^T (\bm{A}^{-1})^T ((1 + 2i\varepsilon_x)(2a/l)^{-i\varepsilon_y}))\bm{A}]^T \times \bm{E} \left[ \bm{A} ((1 + 2i\varepsilon_x)(2a/l)^{-i\varepsilon_y}))\bm{A}^{-1} t^\infty \right].
\] (A.9)

For our problem, we first consider the case of \( \theta = \pi/2 \) in which, the material coordinates \((x_0, y_0)\) is coincident with the fixed spatial coordinates \((x, y)\). According to Eq. (A.9), the energy release rate induced by a remote load \( t^\infty \) can be written as

\[
G_0(t^\infty) = \frac{\pi a}{4} [(t^\infty)^T (\bm{A}_0^{-1})^T ((1 + 2i\varepsilon_x)(2a/l)^{-i\varepsilon_y}))\bm{A}_0] \times \bm{E}_0 \left[ \bm{A}_0 ((1 + 2i\varepsilon_x)(2a/l)^{-i\varepsilon_y}))\bm{A}_0^{-1} t^\infty \right],
\] (A.10)

where subscript “0” stands for quantities referred to the material coordinates \((x_0, y_0)\). With the help of the relevant explicit expressions given by Hwu (1993) for orthotropic bimaterials, the energy release rate (plane strain) of Eq. (A.10) can be explicitly expressed as

\[
G_0(t^\infty) = \frac{\pi a(1 + 4\varepsilon^2)}{4 \cosh^2 \pi \varepsilon} \left[ D_{22}(\sigma_{xy}^\infty)^2 + D_{11}(\sigma_{xy}^\infty)^2 \right]
\] (A.11)

with

\[
e = \frac{1}{2\pi} \ln \frac{1 + \beta}{1 - \beta}, \quad \beta = |W_{21}(D_{11}D_{22})^{-1/2}|,
\] (A.12)

\[
D_{11} = \frac{1}{E_t} \sqrt{\frac{E_t}{E_1}} (1 - v_t^2)^{1/2} \left( \frac{E_t}{\mu} + 2 \left[ \sqrt{(1 - v_t^2)} \left( \frac{E_t}{E_1} - v_t^2 \right) - v_t (1 + v_t) \right] \right)^{1/2},
\] (A.13)

\[
D_{22} = \frac{1}{E_t} \left( 1 - \frac{v_t^2}{E_t} E_t \right)^{1/2} \left( \frac{E_t}{\mu} + 2 \left[ \sqrt{(1 - v_t^2)} \left( \frac{E_t}{E_1} - v_t^2 \right) - v_t (1 + v_t) \right] \right)^{1/2},
\] (A.14)

\[
W_{21} = -\sqrt{\frac{1}{E_t E_1}} \left[ \sqrt{(1 - v_t^2) \left( 1 - \frac{v_t^2}{E_t} E_t \right)} - (1 + v_t) v_t \sqrt{E_t} \right],
\] (A.15)

where we only consider the in-plane load, i.e. \( t^\infty = [\sigma_{xy}^\infty \sigma_{yy}^\infty 0]^T \).

When \( \theta \neq \pi/2 \), the material coordinates \((x_0, y_0)\) does not coincide with the fixed coordinate system \((x, y)\) any more. The transformations of the Barnett–Lothe tensors between these two coordinate systems are

\[
\bm{S} = \bm{Q}\bm{S}_0\bm{Q}^T, \quad \bm{L} = \bm{Q}\bm{L}_0\bm{Q}^T,
\] (A.16)

where \( \bm{Q} \) is the transformation matrix from \((x_0, y_0)\) to \((x, y)\) and

\[
\bm{Q} = \begin{bmatrix} \sin \theta & \cos \theta & 0 \\ -\cos \theta & \sin \theta & 0 \\ 0 & 0 & 1 \end{bmatrix}.
\] (A.17)
Based on (A.16), one can get the transformations formulae of other related matrices as

\[ D = \Omega D_0 \Omega^T, \quad W = \Omega W_0 \Omega^T, \quad M^* = \Omega M_0 \Omega^T, \quad E = \Omega E_0 \Omega^T, \quad \Lambda = \Omega \Lambda_0, \]

where we have used the orthotropic condition of the transformation matrix, \( \Omega^{-1} = \Omega^T \). Substituting the last relation of (A.18) into (A.1), we obtain the stress intensity factor for \( \theta \neq \pi/2 \) as

\[ K = \begin{bmatrix} K_{II} \\ K_I \\ K_{III} \end{bmatrix} = \sqrt{\pi a} \Omega \Lambda_0 \langle (1 + 2i \varepsilon_2)(2a/l)^{-2i} \rangle \Lambda_0^{-1} \Omega^T t^\infty. \]  

(A.19)

The corresponding energy release rate is then

\[ G = \frac{1}{4} K \Lambda^T E K \]

\[ = \frac{\pi a}{4} \left[ t^T \left( \Lambda_0^{-1} \right)^T \langle (1 + 2i \varepsilon_2)(2a/l)^{-2i} \rangle \Lambda_0^T \right] E_0 \left[ \Lambda_0 \langle (1 + 2i \varepsilon_2)(2a/l)^{-2i} \rangle \Lambda_0^{-1} \right], \]

(A.20)

where

\[ \hat{t} = \Omega^T t^\infty = \begin{bmatrix} \sin \theta & - \cos \theta & 0 \\ \cos \theta & \sin \theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \sigma_{xy}^\infty \\ \sigma_{yy}^\infty \\ 0 \end{bmatrix} = \begin{bmatrix} \sigma_{xy}^\infty \sin \theta - \sigma_{yy}^\infty \cos \theta \\ \sigma_{xy}^\infty \cos \theta + \sigma_{yy}^\infty \sin \theta \\ 0 \end{bmatrix}. \]  

(A.21)

Comparing (A.10) with (A.20), the latter can be evaluated simply by replacing \( t^\infty \) with \( \hat{t} \) in the former. If only in-plane load is considered, after some straightforward calculations we obtain

\[ G = \frac{\pi a (1 + 4 \varepsilon^2)}{4 \cosh^2 \frac{\pi e}{2} \varepsilon} \left[ D_{22}(\sigma_{xy}^\infty \cos \theta + \sigma_{yy}^\infty \sin \theta)^2 + D_{11}(\sigma_{xy}^\infty \sin \theta - \sigma_{yy}^\infty \cos \theta)^2 \right], \]

(A.22)

where \( \varepsilon, \ D_{22}, \ D_{11} \) are constants given by Eqs. (A.12)–(A.14). Although Eq. (A.22) is the energy release rate at the right crack tip, similar analysis will lead to an identical expression associated with the left crack tip.

References


